PRACTICAL TOOLS FOR MODELING THE ECONOMIC EFFECTS OF TARIFF CHANGES

David Riker

Samantha Schreiber

ECONOMICS WORKING PAPER SERIES
Working Paper 2020–11–B

U.S. INTERNATIONAL TRADE COMMISSION
500 E Street SW
Washington, DC 20436

November 2020

Practical Tools for Modeling the Economic Effects of Tariff Changes
David Riker and Samantha Schreiber
Economics Working Paper 2020–11–B
November 2020

Abstract

A partial equilibrium model of trade policy will predict different price and quantity effects of tariff changes depending on whether the model is linearized or solved in its non-linear form. Linearized models are usually easier to operate but introduce linear approximation error. After assessing the magnitude of this approximation error, we demonstrate how a multi-step Euler Method approach to solving for the new equilibrium after a tariff change can virtually eliminate approximation error while still producing a modeling tool that is very easy to operate. Spreadsheet versions of both models are available with this paper.

David Riker, Research Division
david.riker@usitc.gov

Samantha Schreiber, Research Division
samantha.schreiber@usitc.gov

1 Introduction

We present a simple, industry-specific model of tariff changes in a single national market supplied by domestic products and imports. The partial equilibrium (PE) model focuses on a single industry without explicit inter-industry spillovers. The model can be used to quantify the incremental effect of the policy change on equilibrium prices, quantities, and revenues from domestic shipments and imports, holding all other factors constant.

Industry-specific PE models are practical tools because they have fewer data requirements than economy-wide models. A model user specifies the magnitude of the tariff change, the market shares prior to the tariff change, and the values of several model parameters representing the responsiveness of supply and demand to changes in prices.

In addition to simpler data requirements, PE models can be much easier to operate if the equations of the models are linearized. In this case, a model user can run simulations in a spreadsheet without a non-linear solver. However, there is a downside to linearization that can offset its benefit: it introduces linear approximation error, especially for large changes in tariff rates.

In this paper, we present non-linear and linearized models, and then we run simulations of hypothetical tariff changes with illustrative data inputs. We compare the estimated price and quantity effects from the two types of models to assess the magnitude of the linear approximation error. As we would expect, linear approximation magnifies and overstates the absolute magnitudes of the price and quantity effects of a tariff change.

Then we introduce a solution technique that sidesteps this trade-off by allowing the user to operate the model in a simple spreadsheet while virtually eliminating approximation error. This multi-step Euler Method approach was developed and applied in Riker (2018) and Riker and Schreiber (2020). It does not require any solvers or statistical software. We provide spreadsheet versions of both the linearized and non-linear Euler Method models along with this paper.

The rest of the paper is presented in five parts. Section 2 presents the non-linear CES tariff model with four different national sources of supply. Section 3 derives a log-linearized version of the model. Section 4 compares simulated effects on prices and quantities using the non-linear and log-linearized models to show the size of the approximation error under different model inputs. Section 5 explains the Euler Method approach to solving the non-linear model. Section 6 concludes.

2 Non-Linear CES Tariff Model

In this section, we describe our PE model of trade and tariffs. Equation (1) is the CES demand curve for products from source country j:

$$q_j = k \; P^{\,\sigma\,-\,1} \; (p_j \; \tau_j)^{-\sigma} \; b_j \tag{1}$$

where q_j is the quantity demanded of the product from source j, and p_j is the producer price of this product. The parameter k is a demand shift term that represents the aggregate expenditure level in the national market, and b_j is a preference asymmetry (or quality differential) term specific to the product from source j. P is the CES industry price index across all sources j. τ_j is a trade cost factor for product from source j. It is equal to one for domestic shipments, and is greater than one for imports. $\sigma > 1$ is the elasticity of substitution between products within the industry, and the elasticity of substitution between industries is one.[1] Equation (2) is the CES price index for the industry.

[1]This reflects a conventional assumption that there are Cobb-Douglas preferences across industry composites.

$$P = \left(\sum_j b_j \ (p_j \ \tau_j)^{1 \ - \ \sigma} \right)^{\frac{1}{1 \ - \ \sigma}} \tag{2}$$

Equation (3) is the supply curve for products from source country j, where a_j is a supply shift parameter, and θ_j is the price elasticity of supply from source j.

$$q_j = a_j \ \ (p_j)^{\theta_j} \tag{3}$$

3 Log-Linearized CES Tariff Model

We totally differentiate (1), (2), and (3) with respect to prices, quantities, and the trade cost factor. Equation (4) is the log-linearized version of the demand curve for products from country j. Hat variables represent percent changes, so $\hat{q}_j = \frac{dq_j}{q_j}$.

$$\hat{q}_j = (\sigma \ - \ 1) \, \hat{P} \ - \ \sigma \, (\hat{p}_j + \hat{\tau}_j) \tag{4}$$

Equation (5) is CES price index for the industry in the national market, analogous to equation (2) in the non-linear model. Equation (6) is the log-linearized version of the supply curve for products from source country j, analogous to equation (3) in the non-linear model.

$$\hat{P} = \sum_j \left(\frac{p_j \ \tau_j \ q_j}{\sum_k p_k \ \tau_k \ q_k} \right) \ (\hat{p}_j \ + \hat{\tau}_j) \tag{5}$$

$$\hat{q}_j = \theta_j \ \hat{p}_j \tag{6}$$

The solution to (4), (5), and (6) is a set of reduced form equations that relate percent changes in the endogenous variables of the model (p_j, q_j, and P) to exogenous changes in the exogenous trade cost factor (τ_j) resulting from changes in the tariff rate on imports from country j.

4 Comparison of Simulations Across Models

The data inputs of the models include the initial expenditures on the products from each source country to the market, initial and revised tariff rates, and estimates of the price responsiveness of demand and supply that are summarized by elasticity values. In this section, we simulate the model using a hypothetical tariff change and illustrative data inputs. In an application to a specific industry and market, the data inputs will be tailored to actual industry data.

We run a series of simulations for a variety of parameter values with both the log-linearized and non-linear models, to show how the estimated effects on the prices, quantities, and revenues of the products vary with the magnitudes of the tariff increase, the elasticity of substitution, the different supply elasticity values, and initial market shares. We assume there are four sources of supply to the national market: three imported sources (a, b, and c) and one domestic source (d). Table 1 reports simulations that all set V_a, V_b, V_c, and V_d equal to 25; θ_a, θ_b, θ_c equal to 10; θ_d equal to 2; and the initial tariff rate on imports from c equal to 0%. There are no tariffs on domestic shipments or on imports from other sources.

The first two columns of estimates in Table 1 report price, quantity, and revenue effects using non-linear and linearized models with benchmark values. All of the estimated effects, both the positive and negative ones, are *overstated* by the linearized model if the true economic relationships are as specified in the non-linear model in equations (1), (2), and (3).

The next two columns in Table 1 compare estimates using non-linear and linear models with a larger tariff increase (10%). All of the effects have the same sign as the benchmark estimates in simulation 1. The estimated effects are not only larger in absolute magnitude, but the ratio of the linearized model estimates to the non-linear model estimates (the linear

Table 1: Illustrative Simulations

Model	Simulation 1.1 Non-Lin	Simulation 1.1 Linear	Simulation 1.2 Non-Lin	Simulation 1.2 Linear	Simulation 1.3 Non-Lin	Simulation 1.3 Linear
Inputs						
Revised Tariff Rate (%)	5	5	**10**	**10**	5	5
Elasticity of Substitution (σ)	5	5	5	5	**7**	**7**
Output (Percent Changes)						
Price of Imports from a	0.32	0.34	0.59	0.68	0.42	0.46
Price of Imports from b	0.32	0.34	0.59	0.68	0.42	0.46
Price of Imports from c	-1.30	-1.33	-2.55	-2.66	-1.58	-1.60
Price of Domestic Product d	0.68	0.72	1.27	1.45	0.80	0.86
Quantity of Imports from a	3.20	3.38	6.07	6.76	4.28	4.56
Quantity of Imports from b	3.20	3.38	6.07	6.76	4.28	4.56
Quantity of Imports from c	-12.29	-13.29	-22.80	-26.57	-14.70	-16.02
Quantity of Domestic Product d	1.36	1.45	2.56	2.90	1.60	1.72
Revenue of Imports from a	3.53	3.72	6.70	7.44	4.72	5.02
Revenue of Imports from b	3.53	3.72	6.70	7.44	4.72	5.02
Revenue of Imports from c	-13.43	-14.61	-24.77	-29.23	-16.04	-17.63
Revenue of Domestic Product d	2.05	2.17	3.86	4.35	2.40	2.59
Consumer Price Index P	1.19	1.27	2.23	2.54	1.20	1.29

approximation error) is increasing in the size of the change in the tariff rate. For example, the ratio of the linear to non-linear percent increases in producer revenue from imports from c is $\frac{14.61}{13.43} = 1.09$ when the benchmark tariff increases to 5% and $\frac{29.23}{24.77} = 1.18$ when the tariff increases to 10%.

The final two columns of estimates in Table 1 return to the benchmark tariff increase (5%) but increase the elasticity of substitution from the benchmark value of 5 to 7. The estimated effects all have the same signs as the benchmark simulations, and they are all greater in absolute magnitude than the benchmark estimates. The extent of linear approximation error is only slightly larger: the ratio of the linear to non-linear percent increase in producer revenue from imports from c is $\frac{14.61}{13.43} = 1.09$ for the benchmark tariff and $\frac{17.63}{16.04} = 1.10$ when the elasticity of substitution is 7 rather than 5.

Table 2 reports additional simulations that vary other model parameters: the domestic and import supply elasticity values and the initial expenditure share on imports. In all of the simulations, we set σ equal to 5, the initial tariff rate on imports from c is 0% and increases to 5%, and there are no tariffs on domestic shipments or imports from other sources. The first two columns of estimates in Table 2 reduce the domestic supply elasticity value, from the benchmark value of 2 to 1. These estimates are very similar to the benchmark estimates in the first two columns of Table 1. The middle two columns shift the initial market shares, reducing the total import market share from 75% to 45% and the share from imports from c from 25% to 15%. The estimated effects are all smaller than the benchmark simulation in absolute magnitude. The linearization magnifies the positive and negative effects of the tariff increase. The last two columns of estimates in Table 2 increase the import supply elasticity values from 10 to 20. This significantly magnifies the reduction in producer revenue of imports from c. The linear approximation error is similar to the benchmark simulation in Table 1.

Table 2: Additional Sensitivity Analyses

	Simulation 2.1		Simulation 2.2		Simulation 2.3	
Model	Non-Lin	Linear	Non-Lin	Linear	Non-Lin	Linear
Inputs						
Domestic Supply Elasticity	**1**	**1**	2	2	2	2
Initial Expenditure on a	25	25	**15**	**15**	25	25
Initial Expenditure on b	25	25	**15**	**15**	25	25
Initial Expenditure on c	25	25	**15**	**15**	25	25
Initial Expenditure on d	25	25	**55**	**55**	25	25
Import Supply Elasticity for a	10	10	10	10	**20**	**20**
Import Supply Elasticity for b	10	10	10	10	**20**	**20**
Import Supply Elasticity for c	10	10	10	10	**20**	**20**
Output (Percent Changes)						
Price of Imports from a	0.33	0.35	0.22	0.24	0.20	0.22
Price of Imports from b	0.33	0.35	0.22	0.24	0.20	0.22
Price of Imports from c	-1.29	-1.32	-1.40	-1.43	-0.77	-0.78
Price of Domestic Product d	0.82	0.88	0.47	0.51	0.72	0.78
Quantity of Imports from a	3.33	3.51	2.21	2.36	4.09	4.34
Quantity of Imports from b	3.33	3.51	2.21	2.36	4.09	4.34
Quantity of Imports from c	-12.18	-13.16	-13.13	-14.31	-14.36	-15.66
Quantity of Domestic Product d	0.82	0.88	0.94	1.01	1.44	1.55
Revenue of Imports from a	3.67	3.86	2.44	2.59	4.30	4.56
Revenue of Imports from b	3.67	3.86	2.44	2.59	4.30	4.56
Revenue of Imports from c	-13.32	-14.47	-14.34	-15.74	-15.02	-16.44
Revenue of Domestic Product d	1.65	1.75	1.42	1.52	2.17	2.33
Consumer Price Index P	1.23	1.32	0.82	0.88	1.26	1.36

5 Multi-Step Euler Method Non-Linear Model

In this section, we introduce a technique for gaining the ease of operation of a linear model without the disadvantages of linear approximation. The Euler Method approach that we use was developed and applied in Riker (2018) and Riker and Schreiber (2020). It simplifies the operation of the model by creating a spreadsheet model that does not require specialized mathematical software with non-linear solvers, like Mathematica, or expert understanding of the underlying equations of the economic models.

The reduced-form equations for the log-linearized model are also the updating formulas in the spreadsheet model. The simulations divide the total percent change in the trade factor on imports from c into many small steps. If the total percent change $\hat{\tau}_c$ is divided into 3000 steps with a constant step-to-step growth rate of g, then $(1 \; + \; \tau_c) = (1 \; + \; g)^{3000}$, and $g = (1 \; + \; \tau_c)^{\frac{1}{3000}} - 1$. In each step of the simulation, equilibrium prices and quantities are re-calculated according to the updating formulas. After the 3000 steps, the cumulative changes in the tariff factor will be equal to the total policy change $\hat{\tau}_c$. The estimates based on the 3000-step Euler method spreadsheet model are identical to non-linear model estimates from non-linear solvers in Mathematica (at least at the level of precision reported in Tables 1 and 2).

The estimated effects reported in Tables 1 and 2 for the non-linear models are all calculated using the Euler method. They can be easily reproduced by altering the inputs of the spreadsheet models.

6 Conclusions

It is possible to make the economic effects of tariff changes easier to calculate without creating unnecessary approximation error. We demonstrate a multi-step Euler Method technique that can be used to estimate non-linear models in simple spreadsheets, while virtually eliminating linear approximation error.

The same approach can be used with more elaborate structural models. Riker and Schreiber (2020) provides a large collection of industry-specific simulation models that use this approach to solve PE models that include a variety of different structural features.

References

Riker, D. (2018). Multinational Production and Employment in an Industry-Specific Model of Trade. U.S. International Trade Commission, Economics Working Paper 2018-08-C.

Riker, D. and Schreiber, S. (2020). Trade Policy PE Modeling Portal. U.S. International Trade Commission. http://www.usitc.gov/data/pe_modeling/index.htm.