Three Regions Tariff Model

D. Riker, 05/13/19 version

This partial equilibrium (PE) model of ad valorem tariffs has three sources of supply and three regional markets, labeled A, B, and C. Consumer demands have a non-nested CES form for the products of the industry. Total industry demand in the region has a constant price elasticity that is region-specific. The supply for each of the three sources has a constant price elasticity. The data inputs of the model are the initial expenditures, initial tariffs, and revised tariffs. The parameter inputs of the model are the elasticity of substitution, the price elasticity of total demand, and the price elasticity of each of the sources of supply. The model also includes demand and supply shift parameters that are calibrated to initial market equilibrium prices and quantities.

The model simulates the effects on prices, quantities, and consumer expenditures of a change in ad valorem tariff rates.

The user can modify data inputs, elasticity values, and tariff rates in the simulation by change the values in the ORANGE-shaded lines in the notebook below tab. The spreadsheet will update the estimated changes in economic outcomes that are reported in the GREEN-shaded cells once the user selects “Evaluate Notebook” under “Evaluation” in the Menu above.

This model is provided as a generic analytical tool, and the data and parameter values are fictional and illustrative. Actual data and parameter values should be supplied by the user based on the industry and market to which the model is applied. The model is the result of ongoing professional research of USITC staff and may be updated. The model is not meant to represent in any way the view of the U.S. International Trade Commission or any of its individual Commissioners. The model is posted to promote the active exchange of ideas between USITC staff and experts outside the USITC and to provide useful economic modeling tools to the public.

In[ ]:=
```
ClearAll[f];
```

Parameter Inputs

Elasticity of Substitution

In[ ]:=
```
sigma = 3;
```

Total Industry Price Elasticity of Demand

In[ ]:=
```
etaA = -1;
```

In[ ]:=
```
etaB = -1;
```

In[ ]:=
```
etaC = -1;
```

Supply Elasticity Parameters

In[ ]:=
```
eA = 5;
```

In[ ]:=
```
eB = 5;
```

In[ ]:=
```
eC = 5;
```

Initial Ad Valorem Tariffs

In[ ]:=
```
tBA0 = 0.10;
```

In[ ]:=
```
tCA0 = 0.00;
```

In[ ]:=
```
tAB0 = 0.00;
```

In[ ]:=
```
tAC0 = 0.00;
```

In[ ]:=
```
tBC0 = 0.00;
```

In[ ]:=
```
tCB0 = 0.00;
```

Revised Ad Valorem Tariff

In[ ]:=
```
tBA = 0.00;
```

In[ ]:=
```
tCA = 0.00;
```

In[ ]:=
```
tAB = 0.00;
```

In[ ]:=
```
tAC = 0.00;
```

In[ ]:=
```
tBC = 0.00;
```

In[ ]:=
```
tCB = 0.00;
```

Initial Equilibrium Values

Expenditures in A

In[ ]:=
```
vAA0 = 100;
```

In[ ]:=
```
vBA0 = 100;
```

In[ ]:=
```
vCA0 = 100;
```

Expenditures in B

In[ ]:=
```
vBB0 = 100;
```

In[ ]:=
```
vAB0 = 100;
```

In[ ]:=
```
vCB0 = 100;
```

Expenditures in C

In[ ]:=
```
vCC0 = 100;
```

In[ ]:=
```
vBC0 = 100;
```

In[ ]:=
```
vAC0 = 100;
```

Prices

In[ ]:= pA0 = 1;

In[ ]:= pB0 = 1;

In[ ]:= pC0 = 1;

Quantities from A

In[ ]:= $qAA0 = \frac{vAA0}{pA0};$

In[ ]:= $qBA0 = \frac{vBA0}{pB0\ (1 + tBA0)};$

In[ ]:= $qCA0 = \frac{vCA0}{pC0\ (1 + tCA0)};$

Quantities from B

In[ ]:= $qBB0 = \frac{vBB0}{pB0};$

In[ ]:= $qAB0 = \frac{vAB0}{pA0\ (1 + tAB0)};$

In[ ]:= $qCB0 = \frac{vCB0}{pC0\ (1 + tCB0)};$

Quantities from C

In[ ]:= $qCC0 = \frac{vCC0}{pC0};$

In[ ]:= $qBC0 = \frac{vBC0}{pB0\ (1 + tBC0)};$

In[ ]:= $qAC0 = \frac{vAC0}{pA0\ (1 + tAC0)};$

Calibration of Parameters Based on the Initial Equilibrium

In[ ]:= $aA = \left(\frac{vAA0}{pA0} + \frac{vAB0}{pA0\ (1 + tAB0)} + \frac{vAC0}{pA0\ (1 + tAC0)}\right) pA0^{-eA};$

In[ ]:= $aB = \left(\frac{vBB0}{pB0} + \frac{vBA0}{pB0\ (1 + tBA0)} + \frac{vBC0}{pB0\ (1 + tBC0)}\right) pB0^{-eB};$

In[ ]:= $aC = \left(\frac{vCC0}{pC0} + \frac{vCA0}{pC0\ (1 + tCA0)} + \frac{vCB0}{pC0\ (1 + tCB0)}\right) pC0^{-eC};$

In[ ]:= $bAB = \frac{vAB0 \left(\frac{pA0\ (1+tAB0)}{pB0}\right)^{sigma-1}}{vBB0};$

In[ ]:= $bAC = \frac{vAC0 \left(\frac{pA0\ (1+tAC0)}{pC0}\right)^{sigma-1}}{vCC0};$

In[ ]:= $bBA = \frac{vBA0 \left(\frac{pB0\ (1+tBA0)}{pA0}\right)^{sigma-1}}{vAA0};$

In[ ]:= $bBC = \frac{vBC0 \left(\frac{pB0\ (1+tBC0)}{pC0}\right)^{sigma-1}}{vCC0};$

In[ ]:= $bCA = \frac{vCA0 \left(\frac{pC0\ (1+tCA0)}{pA0}\right)^{sigma-1}}{vAA0};$

In[ ]:= $bCB = \frac{vCB0 \left(\frac{pC0\ (1+tCB0)}{pB0}\right)^{sigma-1}}{vBB0};$

In[∘]:= $PA0 = \left(pA0^{1-sigma} + bBA\ (pB0\ (1 + tBA0))^{1-sigma} + bCA\ (pC0\ (1 + tCA0))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $PB0 = \left(pB0^{1-sigma} + bAB\ (pA0\ (1 + tAB0))^{1-sigma} + bCB\ (pC0\ (1 + tCB0))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $PC0 = \left(pC0^{1-sigma} + bBC\ (pB0\ (1 + tBC0))^{1-sigma} + bAC\ (pA0\ (1 + tAC0))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $kA = vAA0\ PA0^{-sigma-etaA}\ pA0^{sigma-1};$

In[∘]:= $kB = vBB0\ PB0^{-sigma-etaB}\ pB0^{sigma-1};$

In[∘]:= $kC = vCC0\ PC0^{-sigma-etaC}\ pC0^{sigma-1};$

New Equilibrium Values with Revised Tariff

In[∘]:= $PA = \left(pA^{1-sigma} + bBA\ (pB\ (1 + tBA))^{1-sigma} + bCA\ (pC\ (1 + tCA))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $PB = \left(pB^{1-sigma} + bAB\ (pA\ (1 + tAB))^{1-sigma} + bCB\ (pC\ (1 + tCB))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $PC = \left(pC^{1-sigma} + bBC\ (pB\ (1 + tBC))^{1-sigma} + bAC\ (pA\ (1 + tAC))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $EqnA1 = aA\ pA^{eA} == kA\ PA^{sigma+etaA}\ pA^{-sigma} + kB\ PB^{sigma+etaB}\ (pA\ (1 + tAB))^{-sigma}\ bAB + kC\ PC^{sigma+etaC}\ (pA\ (1 + tAC))^{-sigma}\ bAC;$

In[∘]:= $EqnB1 = aB\ pB^{eB} == kB\ PB^{sigma+etaB}\ pB^{-sigma} + kA\ PA^{sigma+etaA}\ (pB\ (1 + tBA))^{-sigma}\ bBA + kC\ PC^{sigma+etaC}\ (pB\ (1 + tBC))^{-sigma}\ bBC;$

In[∘]:= $EqnC1 = aC\ pC^{eC} == kC\ PC^{sigma+etaC}\ pC^{-sigma} + kA\ PA^{sigma+etaA}\ (pC\ (1 + tCA))^{-sigma}\ bCA + kB\ PB^{sigma+etaB}\ (pC\ (1 + tCB))^{-sigma}\ bCB;$

In[∘]:= `FindRoot[{EqnA1, EqnB1, EqnC1}, {pA, pA0}, {pB, pB0}, {pC, pC0}]`

Out[∘]= `{pA → 0.9977, pB → 1.00969, pC → 0.9977}`

In[∘]:= `pA1 = pA /. %;`

In[∘]:= `pB1 = pB /. %%;`

In[∘]:= `pC1 = pC /. %%%;`

In[∘]:= $PA1 = \left(pA1^{1-sigma} + bBA\ (pB1\ (1 + tBA))^{1-sigma} + bCA\ (pC1\ (1 + tCA))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $PB1 = \left(pB1^{1-sigma} + bAB\ (pA1\ (1 + tAB))^{1-sigma} + bCB\ (pC1\ (1 + tCB))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $PC1 = \left(pC1^{1-sigma} + bBC\ (pB1\ (1 + tBC))^{1-sigma} + bAC\ (pA1\ (1 + tAC))^{1-sigma}\right)^{\frac{1}{1-sigma}};$

In[∘]:= $qAA1 = kA\ PA1^{sigma+etaA}\ pA1^{-sigma};$

In[∘]:= $qAB1 = kB\ PB1^{sigma+etaB}\ (pA1\ (1 + tAB))^{-sigma}\ bAB;$

In[∘]:= $qAC1 = kC\ PC1^{sigma+etaC}\ (pA1\ (1 + tAC))^{-sigma}\ bAC;$

In[∘]:= $qBB1 = kB\ PB1^{sigma+etaB}\ pB1^{-sigma};$

In[∘]:= $qBA1 = kA\ PA1^{sigma+etaA}\ (pB1\ (1 + tBA))^{-sigma}\ bBA;$

In[∘]:= $qBC1 = kC\ PC1^{sigma+etaC}\ (pB1\ (1 + tBC))^{-sigma}\ bBC;$

In[∘]:= $qCC1 = kC\ PC1^{sigma+etaC}\ pC1^{-sigma};$

In[∘]:= $qCB1 = kB\ PB1^{sigma+etaB}\ (pC1\ (1 + tCB))^{-sigma}\ bCB;$

In[∘]:= $qCA1 = kA\ PA1^{sigma+etaA}\ (pC1\ (1 + tCA))^{-sigma}\ bCA;$

Percent Changes in Producer Prices

Supplier A

In[∘]:= $$\frac{(pA1 - pA0)\ 100}{pA0}$$

Out[∘]= -0.230032

Supplier B

In[∘]:= $$\frac{(pB1 - pB0)\ 100}{pB0}$$

Out[∘]= 0.968785

Supplier C

In[∘]:= $$\frac{(pC1 - pC0)\ 100}{pC0}$$

Out[∘]= -0.230032

Percent Changes in Quantities

Domestic Shipments in A

In[∘]:= $$\frac{(qAA1 - qAA0)\ 100}{qAA0}$$

Out[∘]= -5.48559

Exports of A to B

In[∘]:= $\frac{(qAB1 - qAB0)\ 100}{qAB0}$

Out[∘]= 1.02548

Exports of A to C

In[∘]:= $\frac{(qAC1 - qAC0)\ 100}{qAC0}$

Out[∘]= 1.02548

Domestic Shipments in B

In[∘]:= $\frac{(qBB1 - qBB0)\ 100}{qBB0}$

Out[∘]= -2.53044

Exports of B to A

In[∘]:= $\frac{(qBA1 - qBA0)\ 100}{qBA0}$

Out[∘]= 21.3708

Exports of B to C

In[∘]:= $\frac{(qBC1 - qBC0)\ 100}{qBC0}$

Out[∘]= -2.53044

Domestic Shipments in C

In[∘]:= $\frac{(qCC1 - qCC0)\ 100}{qCC0}$

Out[∘]= 1.02548

Exports of C to A

In[∘]:= $\frac{(qCA1 - qCA0)\ 100}{qCA0}$

Out[∘]= -5.48559

Exports of C to B

In[ ]:=
$$\frac{(qCB1 - qCB0)\ 100}{qCB0}$$

Out[ ]= 1.02548

Percent Change in Consumer Expenditures on Imports and Domestic Shipments

Domestic Shipments in A

In[ ]:=
$$\frac{(pA1\ qAA1 - pA0\ qAA0)\ 100}{pA0\ qAA0}$$

Out[ ]= -5.70301

Exports of A to B

In[ ]:=
$$\frac{(pA1\ (1 + tAB)\ qAB1 - pA0\ (1 + tAB0)\ qAB0)\ 100}{pA0\ (1 + tAB0)\ qAB0}$$

Out[ ]= 0.793084

Exports of A to C

In[ ]:=
$$\frac{(pA1\ (1 + tAC)\ qAC1 - pA0\ (1 + tAC0)\ qAC0)\ 100}{pA0\ (1 + tAC0)\ qAC0}$$

Out[ ]= 0.793084

Domestic Shipments in B

In[ ]:=
$$\frac{(pB1\ qBB1 - pB0\ qBB0)\ 100}{pB0\ qBB0}$$

Out[ ]= -1.58617

Exports of B to A

In[ ]:=
$$\frac{(pB1\ (1 + tBA)\ qBA1 - pB0\ (1 + tBA0)\ qBA0)\ 100}{pB0\ (1 + tBA0)\ qBA0}$$

Out[ ]= 11.406

Exports of B to C

In[ ]:=

$$\frac{(pB1\ (1 + tBC)\ qBC1 - pB0\ (1 + tBC0)\ qBC0)\ 100}{pB0\ (1 + tBC0)\ qBC0}$$

Out[ ]= -1.58617

Domestic Shipments in C

In[ ]:=

$$\frac{(pC1\ qCC1 - pC0\ qCC0)\ 100}{pC0\ qCC0}$$

Out[ ]= 0.793084

Exports of C to A

In[ ]:=

$$\frac{(pC1\ (1 + tCA)\ qCA1 - pC0\ (1 + tCA0)\ qCA0)\ 100}{pC0\ (1 + tCA0)\ qCA0}$$

Out[ ]= -5.70301

Exports of C to B

In[ ]:=

$$\frac{(pC1\ (1 + tCB)\ qCB1 - pC0\ (1 + tCB0)\ qCB0)\ 100}{pC0\ (1 + tCB0)\ qCB0}$$

Out[ ]= 0.793084