EULER METHOD PE MODELS OF TARIFF CHANGES

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Abstract

This paper provides technical documentation for a set of industry-specific, partial equilibrium (PE) models of international trade that can be used to simulate the economic impact of changes in tariff rates. First, we identify the data inputs of the models. Then we present the equations underlying the models and the Euler method technique for running the simulations in simple Excel spreadsheets. Finally, we describe four variants of the model currently available in this spreadsheet format.

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1 Introduction

Industry-specific, partial equilibrium (PE) models are sets of equations that identify the economic factors that influence the prices and sales of imports and competing domestic products in the industry. The equations can be used to estimate the impact of changes in tariff rates on prices, domestic shipments, or imports.[1] The models are often used for prospective analysis of policy changes not yet in force, though they can also be used to analyze the impact of policy changes in the past.

The rest of the paper is organized into four sections. The second section identifies the data inputs of the models. The third section presents the equations of the basic model. The fourth section explains the Euler method approach to simulating the economic effects of the policy change. The final section describes four variants of the model currently available in a user-friendly spreadsheet format.

2 Data Inputs of the Models

The data inputs of the models include the initial value of sales of the specific product from each supplier to the market, initial and revised tariff rates, and estimates of the price responsiveness of demand and supply, summarized by elasticity values.[2]

When modeling, it is important to understand the definitions and the limitations of available industry data. PE models are usually designed to reduce data requirements by adopting restrictive assumptions: for example, they might assume that the wage rate that each firm pays is not affected by changes in industry-specific tariff rates if industry employment is only a small share of total labor supply in the economy.

[1] Hallren and Riker (2017) provides an introduction to this type of model.

[2] Hertel, Hummel, Ivanic and Keeney (2017) is an example of an academic study that provides estimates of elasticity values.

3 Equations of the Basic Model

The basic model focuses on a specific industry in a single national market. The model adopts the Armington (1969) assumption that products are differentiated by source country. There are three products sold in the market: domestic shipments (denoted d), imports that are subject to the trade policy change (s), and imports that are not subject to the policy change (n). The producer price of goods from source $j \in \{d, s, n\}$ is p_j, and the equilibrium quantity is q_j.[3] Equilibrium prices and quantities are endogenous variables of the model. The policy variable $\tau_s \geq 1$ is the tariff factor on imports that are subject to the tariff change. It is equal to one plus the ad valorem tariff rate on these imports.[4] τ_s is an exogenous variable of the model.

Consumers have nested constant elasticity of substitution (CES) preferences for the three products. The elasticity of substitution for the two types of imports (θ) is weakly greater than the elasticity of substitution between the domestic product and a CES composite of the imports (σ). Equation (1) is the CES price index for the two types of imports, and equation (2) is the total industry CES price index for the market.

$$PM = \left(b_s \ (p_s \ \tau_s)^{1-\theta} + b_n \ (p_n)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{1}$$

$$P = \left((p_d)^{1-\sigma} + (PM)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{2}$$

b_s and b_n are calibrated parameters that capture preference symmetries and differences in the quality of the different products. The parameter $\eta < 0$ is the price elasticity of total demand in the industry. Equations (3), (4), and (5) are the demand functions for the three products.

[3]Quantity refers to a count of items or a weight measure like tons.
[4]It is not the tariff rate; this trade cost factor is often called the power of the tariff.

$$q_d = k\ (P)^{\eta}\ \left(\frac{p_d}{P}\right)^{-\sigma} \tag{3}$$

$$q_s = k\ b_s\ (P)^{\eta}\ \left(\frac{PM}{P}\right)^{-\sigma}\ \left(\frac{p_s\ \tau_s}{PM}\right)^{-\theta} \tag{4}$$

$$q_n = k\ b_n\ (P)^{\eta}\ \left(\frac{PM}{P}\right)^{-\sigma}\ \left(\frac{p_n}{PM}\right)^{-\theta} \tag{5}$$

b_s and b_n are calibrated parameters that capture preference symmetries and differences in the quality of the different products. k is a demand parameter that is calibrated to the size of market.

Equations (6), (7), and (8) are the supply functions for the three products. They have constant price elasticities ϵ_d, ϵ_s, and ϵ_n.

$$q_d = a_d\ (p_d)^{\epsilon_d} \tag{6}$$

$$q_s = a_s\ (p_s)^{\epsilon_s} \tag{7}$$

$$q_n = a_n\ (p_n)^{\epsilon_n} \tag{8}$$

a_d, a_s, and a_n are calibrated parameters that capture cost factors that remain fixed in the simulations of changes in tariff rates (e.g., wage rates, energy costs).

The market equilibrium is the set of prices for which the demand for each product is equal to its supply. These equations can be used to simulate the change in equilibrium prices and quantities resulting from specific changes in tariff rates. The first step in the simulation is to use the equations of the model, initial tariff rates, and initial prices and quantities in the market to calibrate the a, b, and k parameters. The second step is to use the equations of the model, revised tariff rates, and calibrated parameters to estimate new equilibrium prices and quantities after the policy changes. The final step is to calculate the percentage change from the initial values of the prices and quantities to their new equilibrium values. These percentage changes are estimates of the economic impact of the policy change, holding all other factors fixed.

4 Euler Method for Simulating the Policy Changes

The simulations can be run in mathematical software with non-linear solvers like Mathematica or GAMS. However, they can also be run in more user-friendly Excel spreadsheet using an Euler method approach. The system of equations (1) through (8) can be simplified to the following three equations in prices:

$$a_d \ (p_d)^{\epsilon_d} = k \ (P)^{\eta} \ \left(\frac{p_d}{P}\right)^{-\sigma} \tag{9}$$

$$a_s \ (p_s)^{\epsilon_s} = k \ b_s \ (P)^{\eta} \ \left(\frac{PM}{P}\right)^{-\sigma} \ \left(\frac{p_s \ \tau_s}{PM}\right)^{-\theta} \tag{10}$$

$$a_n \ (p_n)^{\epsilon_n} = k \ b_n \ (P)^{\eta} \ \left(\frac{PM}{P}\right)^{-\sigma} \ \left(\frac{p_n}{PM}\right)^{-\theta} \tag{11}$$

Totally differentiating (1), (2), (9), (10), and (11) with respect to prices and the exogenous tariff factor τ_s results in the following five linear equations in the proportional changes in prices: [5]

[5] The notation $\hat{p}_j$ represents the proportional change in the producer price from source j, $\frac{dp_j}{p_j}$.

$$\hat{PM} = \left(\frac{\tau_s \; p_s \; q_s}{\tau_s \; p_s \; q_s \; + \; p_n \; q_n} \right) \; (\hat{p}_s + \hat{\tau}_s) + \left(\frac{p_n \; q_n}{\tau_s \; p_s \; q_s \; + \; p_n \; q_n} \right) \; \hat{p}_n \tag{12}$$

$$\hat{P} = \left(\frac{p_d \; q_d}{p_s \; q_d \; + \; \tau_s \; p_s \; q_s \; + \; p_n \; q_n} \right) \; \hat{p}_d + \left(\frac{\tau_s \; p_s \; q_s \; + \; p_n \; q_n}{p_s \; q_d \; + \; \tau_s \; p_s \; q_s \; + \; p_n \; q_n} \right) \; \hat{PM} \tag{13}$$

$$\epsilon_d \; \hat{p}_s = (\sigma + \eta) \, \hat{P} \; - \sigma \; \hat{p}_d \tag{14}$$

$$\epsilon_s \; \hat{p}_s = (\sigma + \eta) \, \hat{P} \; - \sigma \; (\hat{p}_s \; + \; \hat{\tau}_s) \tag{15}$$

$$\epsilon_n \; \hat{p}_n = (\sigma + \eta) \, \hat{P} \; - \sigma \; \hat{p}_n \tag{16}$$

The solution to equations (12) through (16) can be represented as a set of reduced-form equations for $\hat{p}_d$, $\hat{p}_s$, and $\hat{p}_n$. These (messy) reduced-form equations are the updating formulas for the spreadsheet models.

The simulation divides the total percentage change in the tariff factor τ_s into many small steps. If the total percentage change $\hat{\tau}_s$ is divided into 3,000 steps with constant step-to-step growth rate g, then $(1 + \hat{\tau}_s) = (1 + g)^{3000}$ and

$$g = (1 + \hat{\tau}_s)^{1/3000} - 1 \tag{17}$$

In each step of the simulation, equilibrium prices are re-calculated according to the updating formulas and quantities are re-calculated according to (6), (7), and (8). After the 3,000 steps, the cumulative changes in the tariff factor will be equal to the total policy change in the simulation, $\hat{\tau}_s$.

The advantage of the Euler method approach is that it results in a spreadsheet model that is easy to operate, without specialized mathematical software or an expert understanding of the underlying equations of the economic model. The limitations of the approach are that this type of spreadsheet model can be time-intensive to create and there are practical limitations on the complexity of the structural equations of the models.[6]

One way to balance these advantages and limitations is to use software like Mathematica or GAMS when developing new models and running simulations based on complex variants of the model, while using Euler method spreadsheets for simulations based on standard models of interest to users who do not build PE models.

5 Variants of the Basic Model

There are several variants of the basic model that have been translated into a user-friendly spreadsheet format. All of the variants described in this paper assume perfect competition in the market, meaning that every producer in the industry is so small that the producer takes the market price as given and does not act strategically.[7]

5.1 Non-Nested CES Model

This is probably the most common model in industry-specific trade policy analysis. This is the model described in (1) to (8). There are differentiated products from three distinct sources of supply. The non-nested model imposes the simplifying restriction that $\sigma = \theta$. The model can simulate the effects of tariff changes on prices and quantities of imports and domestic shipments in the single national market.

[6] The limit is set by the character limit on the cell formulas in Excel. There are also limitations on the numerical precision of most spreadsheet software. Users who are very concerned about numerical precision should consider using specialized mathematical software like Mathematica.

[7] This is a common simplifying assumption in PE models that simulate changes in tariffs.

5.2 Nested CES Model

This is a variant of the model that allows for a higher elasticity of substitution between the two sources or types of imports. The equations of the model again correspond to (1) through (8), but σ is greater than θ. This model can also simulate the effects of tariff changes on prices and quantities of imports and domestic shipments in the market.

5.3 Non-Nested CES with No Domestic Product

This third variant includes two foreign sources of supply to the market, subject imports and non-subject imports, but there is no domestic product. This is equivalent to the Non-Nested CES model except that q_d is equal to zero (or equivalently, a_d is equal to zero). Equations (18) through (22) describe this variant of the model.

$$P = \left(b_s\ (p_s\ \tau_s)^{1-\sigma}\ + b_n\ (p_n\ \tau_n)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{18}$$

$$q_s = k\ (P)^{\eta}\ \left(\frac{p_s\ \tau_s}{P}\right)^{-\sigma}\ b_s \tag{19}$$

$$q_n = k\ (P)^{\eta}\ \left(\frac{p_n\ \tau_n}{P}\right)^{-\sigma}\ b_n \tag{20}$$

$$q_s = a_s\ p_s^{\epsilon_s} \tag{21}$$

$$q_n = a_n\ p_n^{\epsilon_n} \tag{22}$$

This variant can simulate the effects of tariff changes on prices and quantities of the two types of imports in the single market.

5.4 Non-Nested CES Model with Two Distinct Markets

This final variant includes two source countries and two destination markets, domestic (d) and foreign (f). Equations (23) through (28) describe this variant of the model.

$$P_d = \left((p_d)^{1-\sigma} \; + \; b_{fd} \; (p_f \; \tau_{fd})^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{23}$$

$$P_f = \left((p_d \; \tau_{df})^{1-\sigma} \; + \; b_{df} \; (p_f)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{24}$$

$$q_d = k_d \; (P_d)^{\eta_d} \; \left(\frac{p_d}{P_d}\right)^{-\sigma} \; + \; k_f \; (P_f)^{\eta_f} \; \left(\frac{p_d \; \tau_{df}}{P_f}\right)^{-\sigma} \; b_{df} \tag{25}$$

$$q_f = k_d \; (P_d)^{\eta_d} \; \left(\frac{p_f \; \tau_{fd}}{P_d}\right)^{-\sigma} \; b_{fd} \; + \; k_f \; (P_f)^{\eta_f} \; \left(\frac{p_f}{P_f}\right)^{-\sigma} \tag{26}$$

$$q_d = a_d \; p_d^{\epsilon_d} \tag{27}$$

$$q_f = a_f \; p_f^{\epsilon_f} \tag{28}$$

This model can simulate the effects on prices and volumes of imports and domestic production in both of the markets (d and f).

References

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