STRUCTURAL EQUATIONS FOR PE MODELS IN GROUP 3 (FIRM HETEROGENEITY)

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Abstract

This paper presents the structural equations for the third group of industry-specific simulation models of changes in trade policy that are available for download on the USITC's PE Modeling Portal at `https://www.usitc.gov/data/pe_modeling/index.htm`.

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1 Introduction

One of the spreadsheet models incorporates fixed costs, firm heterogeneity, and cross-border trade. The second model extends the first to include foreign affiliate sales.

2 Model with Fixed Costs and Firm Heterogeneity

The first model is a two-country Melitz (2003) model of international trade with firm heterogeneity. The industry-specific model adopts useful simplifications and distributional assumptions from Helpman, Melitz and Yeaple (2004) and Chaney (2008).[1] Within the industry, there is a continuum of firms supplying differentiated products, with constant elasticity of substitution σ. The firms vary in their unit labor requirements. Firm-specific productivity has a Pareto distribution with shape parameter γ.[2] There is a fixed cost of production f_D and an incremental fixed cost of exporting f_X. D is the aggregate value of domestic shipments in the market, integrated over the mass n_D of domestic suppliers.[3]

$$D = k \left(\frac{n_d}{n_d + n_f \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma - 1} + 1}} \right) \tag{1}$$

M is the aggregate value of imports into the market, integrated over the mass n_F of foreign suppliers. k is total industry expenditures in the market, and τ is a variable trade cost on imports.

[1] Khachaturian and Riker (2016) provides a step-by-step derivation of the model, for an extended version that includes FDI.

[2] The model adopts the standard assumption in the literature that $\gamma > \sigma - 1$. DiGiovanni, Levchenko and Rancière (2011) is a source for industry-specific econometric estimates of these parameter values.

[3] Following Chaney (2008), the model assumes that the number of firms that participate in the market is endogenously determined but the numbers of potential market participants, n_d and n_f, are exogenous.

$$M = k \left(\frac{n_f \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1}}{n_d + n_f \ (\tau)^{-\gamma} \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1}} \right) \tag{2}$$

Equation (3) is the ratio of the value of imports to the value of domestic shipments.

$$\frac{M}{D} = (\tau)^{-\gamma} \left(\frac{n_f}{n_d} \right) \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1} \tag{3}$$

Next, we define $Z_0 = \left(\frac{n_f}{n_d} \right) \left(\frac{f_X}{f_D} \right)^{\frac{-\gamma}{\sigma-1}+1}$. The model calibrates Z_0 based on the ratio of the value of imports to the value of domestic shipments in the initial equilibrium and initial trade costs.

$$Z_0 = \left(\frac{M_0}{D_0} \right) \ (\tau_0)^{\gamma} \tag{4}$$

τ_0 is the initial trade cost factor, M_0 is the initial value of imports, and D_0 is the initial value of domestic shipments. An increase in the fixed cost of exporting f_X decreases Z and relative expenditure on imports, and an increase in the fixed cost of domestic production f_D increases Z and relative expenditure on imports.

$$Z = Z_0 \ \left(1 + \left(\frac{-\gamma}{\sigma - 1} + 1 \right) \left(\left(\frac{f_X - f_{X0}}{f_{X0}} \right) - \left(\frac{f_D - f_{D0}}{f_{D0}} \right) \right) \right) \tag{5}$$

The model simulates the effects of changes in the fixed costs (f_X and f_D) and the variable trade cost (τ) on the value of imports (M) and the value of domestic shipments (D), based on (6) and (7).

$$D = D_0 \ \left(\frac{1 + Z_0 \ (\tau_0)^{-\gamma}}{1 + Z \ (\tau)^{-\gamma}} \right) \tag{6}$$

$$M = M_0 \ \left(\frac{1 + Z_0 \ (\tau_0)^{-\gamma}}{1 + Z \ (\tau)^{-\gamma}} \right) \ \left(\frac{Z(\tau)^{-\gamma}}{Z_0(\tau_0)^{-\gamma}} \right) \tag{7}$$

3 Model with Foreign Affiliate Sales

The second model is based on Helpman et al. (2004) as modified in Khachaturian and Riker (2016). Like the first model, Z_{P0} and Z_{X0} are calibrated to the initial equilibrium in the market.

$$Z_{P0} = \left(\frac{n_f}{n_d}\right)\left(\frac{f_P}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1} = \left(\frac{A_0}{D_0}\right)\left(1 - {C_0}^{1-\sigma}\right)^{-\frac{\gamma}{\sigma-1}+1} \tag{8}$$

$$Z_{X0} = \left(\frac{n_f}{n_d}\right)\left(\frac{f_X}{f_D}\right)^{\frac{-\gamma}{\sigma-1}+1} = \left(\frac{M_0}{D_0}\right){C_0}^{\gamma} + {C_0}^{1-\sigma+\gamma}\left(1 - {C_0}^{1-\sigma}\right)^{\frac{\gamma}{\sigma-1}-1} Z_{P0} \tag{9}$$

The variables A_0, M_0, and D_0 represent the initial values of foreign affiliate sales, cross-border imports, and domestic sales. C_0 is the initial relative variable cost of delivering foreign services supplied to the domestic market, including variable international trade costs. f_P is the incremental fixed cost of foreign affiliate supply, f_X is the fixed cost of cross-border trade, and f_D is the fixed cost of provision by domestic suppliers. n_d and n_f are the number of domestic and foreign firms that can potentially supply the domestic market.[4] Again, σ is the elasticity of substitution, and γ is the shape parameter of the Pareto distribution of firm-specific productivity levels.

Changes in the fixed costs of trade affect Z_P and Z_X. Equations (10) and (11) are the updating equations.

$$\frac{Z_P - Z_{P0}}{Z_{P0}} = \left(1 + \left(\frac{-\gamma}{\sigma-1} + 1\right)\left(\left(\frac{f_P - f_{P0}}{f_{P0}}\right) - \left(\frac{f_D - f_{D0}}{f_{D0}}\right)\right)\right) \tag{10}$$

$$\frac{Z_X - Z_{X0}}{Z_{X0}} = \left(1 + \left(\frac{-\gamma}{\sigma-1} + 1\right)\left(\left(\frac{f_X - f_{X0}}{f_{X0}}\right) - \left(\frac{f_D - f_{D0}}{f_{D0}}\right)\right)\right) \tag{11}$$

[4] n_d and n_f are treated as exogenous variables in the partial equilibrium model.

The equilibrium values of foreign affiliate sales (A), cross-border imports (M), and domestic sales (D) are defined by (12), (13), and (14).

$$A = \frac{E\ Z_P\ (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}-1}}{Z_P\ (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}}\ +\ Z_X\ C^{-\gamma}\ + 1} \tag{12}$$

$$M = \frac{E\ C^{1-\sigma}\left(Z_X\ C^{-\gamma+\sigma-1} - Z_P\,(1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}-1}\right)}{Z_P\ (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}}\ +\ Z_X\ C^{-\gamma}\ + 1} \tag{13}$$

$$D = \frac{E}{Z_P\ (1 - C^{1-\sigma})^{\frac{\gamma}{\sigma-1}}\ +\ Z_X\ C^{-\gamma}\ + 1} \tag{14}$$

These three values sum to total expenditure on the industry in the market, E, which is held constant in model simulations.

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