

AN EULER METHOD APPROACH TO SIMULATING CHANGES IN TRADE POLICY

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Abstract

This paper provides technical documentation for a set of industry-specific, partial equilibrium (PE) models of that can be used to simulate the economic impact of changes in trade policies. First, we identify the data inputs of the basic model. Then we present the equations underlying the model and the Euler method technique that we use to run simulations in simple Excel spreadsheets.

The models described in this documentation are the result of ongoing professional research of USITC staff and are solely meant to represent the professional research of individual authors. These papers are not meant to represent in any way the views of the U.S. International Trade Commission or any of its individual Commissioners. Please address correspondence to david.riker@usitc.gov.

1 Introduction

Industry-specific, partial equilibrium (PE) models are sets of equations that identify the economic determinants of prices and sales of imports and competing domestic products. The equations can be used to estimate the impact of changes in trade policy on prices, domestic shipments, and imports.¹ The models are often used for prospective analysis of policy changes not yet in force, though they can also be used to analyze the impact of policy changes in the past.

The rest of this paper is organized into four sections. The second section identifies the data inputs of the basic model. The third section presents the equations of the model. The fourth section explains the Euler method technique that we use to run simulations in simple Excel spreadsheets. The final section discusses the USITC's PE modeling portal, a website with downloadable PE models of trade policy that are available in the user-friendly spreadsheet format.

2 Data Inputs of the Model

The data inputs of the models include the initial value of sales of the specific product from each supplier to the market, initial and revised tariff rates, and estimates of the price responsiveness of demand and supply, summarized by elasticity values.²

When modeling, it is important to understand the definitions and the limitations of available industry data. PE models are usually designed to reduce data requirements by adopting restrictive assumptions: for example, they might assume that the wage rate that each firm pays is not affected by changes in industry-specific tariff rates since employment in the specific industry is only a small share of total labor supply in the entire economy.

¹Hallren and Riker (2017) provides an introduction to this type of PE model.

²Hertel, Hummel, Ivanic and Keeney (2017) is one of many academic studies that provide econometric estimates of these elasticity values.

3 Equations of the Model

The basic model focuses on a specific industry in a single national market. The model adopts the Armington (1969) assumption that products are differentiated by source country. There are three products sold in the market: domestic shipments (denoted d), imports that are subject to the trade policy change (s), and imports that are not subject to the policy change (n). The producer price of goods from source $j \in \{d, s, n\}$ is p_j , and the equilibrium quantity is q_j .³ Equilibrium prices and quantities are endogenous variables of the model. The policy variable $\tau_s \geq 1$ is the tariff factor on the imports that are subject to the tariff change. It is equal to one plus the ad valorem tariff rate on these imports.⁴ τ_s is an exogenous variable of the model.

Consumers have nested constant elasticity of substitution (CES) preferences for the three products. The elasticity of substitution between the two types of imports (θ) is greater than or equal to the elasticity of substitution between the domestic product and a CES composite of the imports (σ). Equation (1) is the CES price index for the two types of imports, and equation (2) is the total industry CES price index for the market.

$$I = (b_s (p_s \tau_s)^{1-\theta} + b_n (p_n)^{1-\theta})^{\frac{1}{1-\theta}} \quad (1)$$

$$P = ((p_d)^{1-\sigma} + (I)^{1-\sigma})^{\frac{1}{1-\sigma}} \quad (2)$$

b_s and b_n are calibrated parameters that capture preference symmetries and differences in the quality of the three products. The parameter $\eta < 0$ is the price elasticity of total demand in the industry. Equations (3), (4), and (5) are the demand functions for the three products.

³Quantity refers to a count of items or a weight measure like tons, not a dollar value.

⁴It is not the tariff rate; this trade cost factor is often called the power of the tariff.

$$q_d = k (P)^\eta \left(\frac{p_d}{P} \right)^{-\sigma} \quad (3)$$

$$q_s = k b_s (P)^\eta \left(\frac{I}{P} \right)^{-\sigma} \left(\frac{p_s \tau_s}{I} \right)^{-\theta} \quad (4)$$

$$q_n = k b_n (P)^\eta \left(\frac{I}{P} \right)^{-\sigma} \left(\frac{p_n}{I} \right)^{-\theta} \quad (5)$$

k is a demand parameter that is calibrated to the size of market.

Equations (6), (7), and (8) are the supply functions for the three products. They have constant price elasticities ϵ_d , ϵ_s , and ϵ_n .

$$q_d = a_d (p_d)^{\epsilon_d} \quad (6)$$

$$q_s = a_s (p_s)^{\epsilon_s} \quad (7)$$

$$q_n = a_n (p_n)^{\epsilon_n} \quad (8)$$

a_d , a_s , and a_n are calibrated parameters that reflect cost factors that are not affected by the changes in tariff rates (e.g., energy costs, possibly wage rates).

The market equilibrium is the set of prices for which the demand for each product is equal to its supply. These eight equations are used to simulate the change in equilibrium prices and quantities resulting from changes in tariff rates. The first step in the simulations is to use the equations of the model and initial tariff rates, prices, and quantities in the market to calibrate the a , b , and k parameters. The second step is to use the equations of the model, revised tariff rates, and calibrated parameters to estimate new equilibrium

prices and quantities that result from the policy changes. The final step is to calculate the percentage change from the initial values of the prices and quantities to their new equilibrium values. These percentage changes are estimates of the economic impact of the policy change, holding all other factors fixed.

4 Euler Method for Simulating the Policy Changes

The simulations can be run in mathematical software that includes a non-linear solver, like Mathematica or GAMS. However, they can also be run in Excel spreadsheets using an Euler method technique to run the simulations. The system of eight equations can be simplified to the following three equations in prices:

$$a_d (p_d)^{\epsilon_d} = k (P)^\eta \left(\frac{p_d}{P} \right)^{-\sigma} \quad (9)$$

$$a_s (p_s)^{\epsilon_s} = k b_s (P)^\eta \left(\frac{I}{P} \right)^{-\sigma} \left(\frac{p_s \tau_s}{I} \right)^{-\theta} \quad (10)$$

$$a_n (p_n)^{\epsilon_n} = k b_n (P)^\eta \left(\frac{I}{P} \right)^{-\sigma} \left(\frac{p_n}{I} \right)^{-\theta} \quad (11)$$

Totally differentiating (1), (2), (9), (10), and (11) with respect to prices and the exogenous tariff factor τ_s results in the following five linear equations in the proportional changes in prices: ⁵

$$\hat{I} = \left(\frac{\tau_s p_s q_s}{\tau_s p_s q_s + p_n q_n} \right) (\hat{p}_s + \hat{\tau}_s) + \left(\frac{p_n q_n}{\tau_s p_s q_s + p_n q_n} \right) \hat{p}_n \quad (12)$$

⁵The notation \hat{p}_j represents the proportional change in the producer price from source j , $\frac{dp_j}{p_j}$.

$$\hat{P} = \left(\frac{p_d q_d}{p_s q_d + \tau_s p_s q_s + p_n q_n} \right) \hat{p}_d + \left(\frac{\tau_s p_s q_s + p_n q_n}{p_s q_d + \tau_s p_s q_s + p_n q_n} \right) \hat{I} \quad (13)$$

$$\epsilon_d \hat{p}_s = (\sigma + \eta) \hat{P} + (\theta - \sigma) \hat{I} - \theta \hat{p}_d \quad (14)$$

$$\epsilon_s \hat{p}_s = (\sigma + \eta) \hat{P} + (\theta - \sigma) \hat{I} - \theta (\hat{p}_s + \hat{\tau}_s) \quad (15)$$

$$\epsilon_n \hat{p}_n = (\sigma + \eta) \hat{P} - \sigma \hat{p}_n \quad (16)$$

The solution to (12) through (16) can be represented as a set of reduced-form equations for \hat{p}_d , \hat{p}_s , and \hat{p}_n . These reduced-form equations define the updating formulas in the spreadsheet models.

The simulations divide the total percentage change in the tariff factor τ_s into many small steps. If the total percentage change $\hat{\tau}_s$ is divided into 3,000 steps with constant step-to-step growth rate g , then $(1 + \hat{\tau}_s) = (1 + g)^{3000}$ and

$$g = (1 + \hat{\tau}_s)^{1/3000} - 1 \quad (17)$$

In each step of the simulation, equilibrium prices are re-calculated according to the updating formulas and quantities are re-calculated according to (6), (7), and (8). After the 3,000 steps, the cumulative changes in the tariff factor will be equal to the total policy change $\hat{\tau}_s$.

The advantage of the Euler method technique is that it creates spreadsheet models that are easy to operate, without specialized mathematical software or an expert understanding of the underlying equations of the economic models. The limitations of the approach are that it can be time-consuming to create the spreadsheet models, and there are practical limitations on the complexity of the structural equations of the models.

One way to balance these advantages and limitations is to use software like Mathematica or GAMS when developing new models and running simulations based on complex versions of the model, while using Euler method spreadsheets for simulations based on standard models commonly used by policy analysts who do not build their own PE models.

5 Variations on the Basic Model

There are many different types of PE models of changes in trade policy, and several have been translated into Euler method spreadsheets. These are available for download at the USITC's PE Modeling Portal at https://www.usitc.gov/data/pe_modeling/index.htm.

The Portal also includes spreadsheet models in which the simulated economic outcomes have closed-form solutions and it is not necessary to use the Euler method in the simulation. For example, this is the case for the Portal spreadsheets based on monopolistic competition models.

We expect to continue to expand the set of models available and to periodically update the Portal.

References

- Armington, P. S. (1969). A Theory of Demand for Products Distinguished by Place of Production., *Staff Papers (International Monetary Fund)* **16**(1): 159–78.
- Hallren, R. and Riker, D. (2017). An Introduction to Partial Equilibrium Modeling of Trade Policy, *U.S. International Trade Commission Economics Working Paper No. 2017-07-B*.
- Hertel, T., Hummel, D., Ivanic, M. and Keeney, R. (2017). How Confident Can We Be of CGE-Based Assessments of Free Trade Agreements?, *Economic Modeling* **24**(4): 611–35.