STRUCTURAL EQUATIONS FOR PE MODELS IN GROUP 1 (PERFECT COMPETITION)

David Riker and Samantha Schreiber

U.S. International Trade Commission, Office of Economics

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Abstract

This paper presents the structural equations for the first group of Euler method simulation models of changes in trade policy that are available for download on the USITC's PE Modeling Portal at `https://www.usitc.gov/data/pe_modeling/index.htm`.

 Please address correspondence to david.riker@usitc.gov.

1 Basic Modeling Framework

The basic model focuses on a specific industry in a single national market. Products are differentiated by source country. There are three products sold in the market: domestic shipments (denoted d), imports that are subject to the trade policy change (s), and imports that are not subject to the policy change (n). The producer price of goods from source $j \in \{d, s, n\}$ is p_j, and the equilibrium quantity is q_j. The policy variable $\tau_s \geq 1$ is the tariff factor on the imports that are subject to the tariff change. It is equal to one plus the ad valorem tariff rate on these imports.

Consumers have nested constant elasticity of substitution (CES) preferences for the three products. The elasticity of substitution between the two types of imports (θ) is greater than or equal to the elasticity of substitution between the domestic product and a CES composite of the imports (σ). Equation (1) is the CES price index for the two types of imports, and equation (2) is the total industry CES price index for the market.

$$I = \left(b_s \ (p_s \ \tau_s)^{1-\theta} + b_n \ (p_n)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{1}$$

$$P = \left((p_d)^{1-\sigma} + (I)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{2}$$

b_s and b_n are calibrated parameters that capture preference symmetries and differences in the quality of the three products. The parameter $\eta < 0$ is the price elasticity of total demand in the industry. Equations (3), (4), and (5) are the demand functions for the three products.

$$q_d = k \ (P)^{\eta} \ \left(\frac{p_d}{P}\right)^{-\sigma} \tag{3}$$

$$q_s = k \ b_s \ (P)^{\eta} \ \left(\frac{I}{P}\right)^{-\sigma} \ \left(\frac{p_s \ \tau_s}{I}\right)^{-\theta} \tag{4}$$

$$q_n = k\ b_n\ (P)^{\eta}\ \left(\frac{I}{P}\right)^{-\sigma}\ \left(\frac{p_n}{I}\right)^{-\theta} \tag{5}$$

k is a demand parameter that is calibrated to the size of market.

Equations (6), (7), and (8) are the supply functions for the three products. They have constant price elasticities ϵ_d, ϵ_s, and ϵ_n.

$$q_d = a_d\ (p_d)^{\epsilon_d} \tag{6}$$

$$q_s = a_s\ (p_s)^{\epsilon_s} \tag{7}$$

$$q_n = a_n\ (p_n)^{\epsilon_n} \tag{8}$$

a_d, a_s, and a_n are calibrated supply parameters.

The market equilibrium is the set of prices for which the demand for each product is equal to its supply. These eight equations are used to simulate the change in equilibrium prices and quantities resulting from changes in tariff rates.

2 Variations on the Basic Model

There are several variations on the basic model that have been translated into a user-friendly spreadsheet format. All of the variations described in this paper assume perfect competition in the market, meaning that every producer in the industry is so small relative to the size of the whole market that the producer takes the market price as given and does not act strategically.[1]

[1]This is a common simplifying assumption in industry-specific models of changes in trade policy.

2.1 Tariff Model with Non-Nested CES Demand and Three Sources of Supply

This is probably the most common industry-specific model used in trade policy analysis. This is the model described in (1) to (8). There are differentiated products from three distinct sources of supply. Non-nested CES demand means $\sigma = \theta$. The model simulates the effects of tariff changes on prices and quantities of imports and domestic shipments in a single national market.

2.2 Tariff Model with Nested CES Demand and Three Sources of Supply

This model permits the user to specify a higher elasticity of substitution between the two sources or types of imports. With three sources of supply, there are three alternative nesting scenarios: subject imports with non-subject imports, domestic products with subject imports, and domestic products with non-subject imports.

The equations of the model with nesting of subject imports and non-subject imports again correspond to (1) through (8), and σ is less than θ.

The equations of the model with the second nesting scenario, grouping together domestic products and subject imports, replace (1) through (5) with (9) through (13), and again σ is less than θ.

$$I = \left(b_s\ (p_s\ \tau_s)^{1-\theta} + (p_d)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{9}$$

$$P = \left(b_n\ (p_n)^{1-\sigma} + (I)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{10}$$

$$q_d = k\ (P)^{\eta}\ \left(\frac{I}{P}\right)^{-\sigma}\ \left(\frac{p_d}{I}\right)^{-\theta} \tag{11}$$

$$q_s = k\ b_s\ (P)^{\eta}\ \left(\frac{I}{P}\right)^{-\sigma}\ \left(\frac{p_s\ \tau_s}{I}\right)^{-\theta} \tag{12}$$

$$q_n = k\ b_n\ (P)^{\eta}\ \left(\frac{p_n}{I}\right)^{-\sigma} \tag{13}$$

The equations of the model with the third nesting scenario, grouping together domestic products and non-subject imports replace (1) through (5) with (14) through (18), and σ is less than θ.

$$I = \left((p_d)^{1-\theta} + b_n\ (p_n)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{14}$$

$$P = \left(b_s\ (p_s\ \tau_s)^{1-\sigma} + (I)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{15}$$

$$q_d = k\ (P)^{\eta}\ \left(\frac{I}{P}\right)^{-\sigma}\ \left(\frac{p_d}{I}\right)^{-\theta} \tag{16}$$

$$q_s = k\ b_s\ (P)^{\eta}\ \left(\frac{p_s\ \tau_s}{P}\right)^{-\sigma} \tag{17}$$

$$q_n = k\ b_n\ (P)^{\eta}\ \left(\frac{I}{P}\right)^{-\sigma}\ \left(\frac{p_n}{I}\right)^{-\theta} \tag{18}$$

2.3 Tariff Model with Non-Nested CES Demand and No Domestic Production

This model also includes two foreign sources of supply to the market, subject imports and non-subject imports, but in this case there is no domestic product. This is equivalent to the tariff model with non-nested CES demand with three sources of supply except that q_d is equal to zero, or equivalently a_d is equal to zero. Equations (19) through (23) describe this simpler model.

$$P = \left(b_s\ (p_s\ \tau_s)^{1-\sigma}\ + b_n\ (p_n)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{19}$$

$$q_s = k\ (P)^{\eta}\ \left(\frac{p_s\ \tau_s}{P}\right)^{-\sigma}\ b_s \tag{20}$$

$$q_n = k\ (P)^{\eta}\ \left(\frac{p_n}{P}\right)^{-\sigma}\ b_n \tag{21}$$

$$q_s = a_s\ (p_s)^{\epsilon_s} \tag{22}$$

$$q_n = a_n\ (p_n)^{\epsilon_n} \tag{23}$$

This model simulates the effects of tariff changes on prices and quantities of the two types of imports in the single national market. It assumes that there will continue to be no domestic product regardless of the tariff change.

2.4 Tariff Model with Non-Nested CES Demand and Intermediate Imports

This model includes two source countries, domestic (d) and foreign (f). An intermediate input is combined with value added to produce a final good. Country d imports both final and intermediate goods from country f. Equations (24) through (35) describe this model.

Equation (24) is a price index for the upstream product, the intermediate inputs produced in countries d and f. The elasticity of substitution is θ, and the asymmetry parameter is γ.

$$u = \left((u_d)^{1-\theta} \; + \; \gamma \; (u_f \; \tau_{up})^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{24}$$

The price of the domestic product is a price index that combines the cost of the intermediates and value added with price w. The elasticity of substitution is λ, and the asymmetry parameter is α. τ_{up} is equal to one plus the tariff rate on the imported intermediate good.

$$p_d = \left((w)^{1-\lambda} \; + \; \alpha \; (u)^{1-\lambda} \right)^{\frac{1}{1-\lambda}} \tag{25}$$

The model assumes that the supplies of these inputs are perfectly elastic, so u_d, u_f, and w are exogenous variables in the model. Equation (26) is a price index for the downstream product, the final goods produced in countries d and f. The elasticity of substitution is σ, and the asymmetry parameter is β.

$$P = \left((p_d)^{1-\sigma} \; + \; \beta \; (p_f \; \tau_{down})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{26}$$

τ_{down} is equal to one plus the tariff rate on the imported final good. Equations (27) and (28) are the demands for the domestic and imported final goods.

$$q_d = k \; (P)^{\eta} \; \left(\frac{p_d}{P} \right)^{-\sigma} \tag{27}$$

$$q_f = k\ (P)^{\eta}\ \left(\frac{p_f\ \tau_{down}}{P}\right)^{-\sigma}\ \beta \tag{28}$$

Finally, (29) is the derived demand for the imported intermediate input.

$$qint_f = q_d\ p_d \left(\frac{u_f\ \tau_{up}}{u}\right)^{1-\theta} \gamma \tag{29}$$

2.5 Binding Import Quota Model with Nested CES Demand

This model has the same structural equations as the basic model described in (1) through (8), except that (7) is replaced by the assumption that there is a perfectly inelastic supply of subject imports, q_s, that is determined by an exogenous binding import quota. Since this variation of the model focuses on binding quotas rather than tariffs, τ_s is set equal to one. The model simulates the effects of changes in the import quota on prices and quantities of imports and domestic shipments in the market.

2.6 Tariff Model with Minimally Restricted Log-Linear Demand

Equations (30) through (32) are the log-linear demand equations for the model.

$$q_d = k\ {p_d}^{-\lambda_{dd}}\ (p_s\ \tau_s)^{\lambda_{ds}}\ {p_n}^{\lambda_{dn}} \tag{30}$$

$$q_s = k\ b_s\ {p_d}^{\lambda_{sd}}\ (p_s\ \tau_s)^{-\lambda_{ss}}\ {p_n}^{\lambda_{sn}} \tag{31}$$

$$q_n = k\ b_n\ {p_d}^{\lambda_{nd}}\ (p_s\ \tau_s)^{\lambda_{ns}}\ {p_n}^{-\lambda_{nn}} \tag{32}$$

Equations (6) through (8) are still the log-linear supply equations of this final model.

2.7 Tariff Rate Quota Model

This model has the same structural equations as the basic model described in (1) through (8). The difference is that the tariff rate τ_s depends on the level of imports. Defining Q_s as the tariff rate quota (TRQ) quantity, τ_s is equal to the in-quota rate for $q_s \leq Q_s$ and is equal to the out-of-quota rate for $q_s > Q_s$. The model simulates the effects of changes in the TRQ on prices and quantities of imports and domestic shipments in the market.

2.8 Tariff Model with Incomplete Capacity Utilization

This model has the same demand equations as the basic model described in (1) through (5). Supply equations (7) and (8) are replaced with the assumption that the supply of subject and non-subject imports is perfectly elastic at p_s and p_n, and (6) is replaced by the supply function in (33) for domestic production $q_d \leq Q_d$, where Q_d is maximum production capacity in the domestic industry. The model simulates the effects of changes in the tariff on prices and quantities of imports and domestic shipments in the market, as well as the rate of domestic capacity utilization.

$$q_d = Q_d \; - \; \frac{a_d}{p_d} \tag{33}$$

2.9 Entry of a New Source of Imports

The fifth variant addresses the entry of new sources of imports. There are initially three sources of supply to the market, one domestic source (x) and two foreign sources (y and z). Consumers have CES preferences, with elasticity of substitution σ and a price elasticity of total industry demand equal to η. There is perfect competition in the market, and the supply from all three sources is perfectly elastic (i.e., there are no capacity constraints on production), so $p_j = c_j$ for $j \in \{x, y, z\}$.

Equation (22) is the original CES price index for the market, and (23) is the initial equilibrium demand for the product from source $j \in \{x, y, z\}$.

$$P_0 = \left((p_{x0})^{1-\sigma} + b_y\ (p_{y0}\ \tau_{y0})^{1-\sigma} + b_z\ (p_{z0}\ \tau_{z0})^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{34}$$

$$q_{j0} = k\ (P_0)^{\sigma+\eta}\ (p_{j0}\ \tau_{j0})^{-\sigma}\ b_j \tag{35}$$

Equations (24) through (26) calibrate the three demand parameters to initial expenditures and tariff rates, normalizing initial prices to one without loss of generality.

$$b_y = \left(\frac{v_{y0}}{v_{x0}}\right) (\tau_{y0})^{\sigma-1} \tag{36}$$

$$b_z = \left(\frac{v_{z0}}{v_{x0}}\right) (\tau_{z0})^{\sigma-1} \tag{37}$$

$$k = v_{x0}\ \left(1 + b_y\ (\tau_{y0})^{1-\sigma} + b_z\ (\tau_{z0})^{1-\sigma}\right)^{\frac{-\sigma-\eta}{1-\sigma}} \tag{38}$$

Entry leads to a fourth source of supply, entrant e. The model assumes that source z is an appropriate reference group, meaning that the production costs and perceived quality of the products of the new entrant (c_e and b_e) are the same as those of the reference group (c_z and b_z). This is not a model of endogenous entry; it is modeling the economic impact conditional on entry.

Equation (27) is the new equilibrium CES price index that includes source e.

$$P = \left((p_x)^{1-\sigma} + b_y\ (p_y\ \tau_y)^{1-\sigma} + b_z\ (p_z\ \tau_z)^{1-\sigma} + b_e\ (p_e\ \tau_e)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{39}$$

Since supply from each source is perfectly elastic, $p_j = p_{j0} = c_j$ for $j \in \{x, y, z, e\}$. Equation

(14) is the new equilibrium quantity from source j.

$$q_j = k\ (P)^{\sigma+\eta}\ (p_j\ \tau_j)^{-\sigma}\ b_j \tag{40}$$

Finally, (29) is the new equilibrium market share of source j.

$$m_j = \frac{b_j\ (p_j\ \tau_j)^{1-\sigma}}{(p_x)^{1-\sigma} + b_y\ (p_y\ \tau_y)^{1-\sigma} + b_z\ (p_z\ \tau_z)^{1-\sigma} + b_e\ (p_e\ \tau_e)^{1-\sigma}} \tag{41}$$

The model can be used to simulate the entry of a new source of imports, for example due to the reduction or removal of a prohibitive tariff on imports from the new source.