Local Transmission of Trade Shocks

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Abstract

This paper studies theoretically and empirically the geographic transmission of trade shocks over the territory of a country. Increases in labor demand in a location raise local wages and draw workers away from employment in neighboring locations: those locations experience a reduction in labor supply and an increase in prevailing wages even if not initially affected, or not engaged in the production of tradeable goods; adjustment in their wages affect in turn other closeby locations. In addition, increases in prevailing wages in a location affect all the industries producing there: other locations active in the same industries gain then market shares and experience an increase in labor demand even when they are far apart. I develop a model capable of incorporating realistic geographic features and isolate theoretically the different components of this diffusion. The model is general enough to also allow the study of transmission of localized immigration and productivity shocks. I estimate its main components with data on US commuting patterns and sectoral employment. I illustrate the impact of reductions in trade frictions in a sector on locations active and inactive in it, and the consequences of productivity growth on nominal wages of workers vs. real wages of residents. The model delivers insights on the consequences of ignoring commuting flows in analyses of local labor markets.

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1 Introduction

How does a trade shock unfold over the territory of a country, from the locations originally impacted to neighboring places, and through the entire nation? Are locations mainly specialized in the production of non-tradeable goods insulated from the consequences of trade shocks? Understanding the micro-economic details of the transmission mechanisms of international trade shocks is crucial for assessing the within-country and distributional consequences of tariff changes, trade rebalancing, or world-wide productivity growth.

This paper takes as given the distribution of residents and active industries across space, and analyzes the interdependent roles of workers' commuting patterns, on one side, and sectoral concentration and location specialization, on the other, in the diffusion of shocks across a country. If, as a consequence of a trade liberalization in a sector, labor demand increases in a sector-location, growing wages may induce workers commuting to neighboring areas to seek employment in the first location. This adjustment, while limiting the increase in wages where the shock first hits, reduces labor supply in adjacent regions, thus inducing an upward pressure on wages elsewhere: the shock then unfolds through the territory to closeby labor markets. If other sectors are also active in this location, the increase in wage will make them less competitive against producers of the same goods located elsewhere in the country, thus raising labor demand and wages to possibly farther labor markets.

This paper develops a theoretical model capable of studying these effects. The economy is a set of locations where people live and production in several sectors takes place. Agents (exogenously) reside in distinct locations and they can work either where they live or in neighboring places within commuting distance. Residents in a location are always arbitraging between wages paid in any pair of possible commuting destinations: hence, changes in the prevailing wage in the two-neighborhood of a location (not only its neighborhood) can impact labor supply to this location directly; in other words, absence of commuting flows does not imply absence of linkages across labor markets. I will say that two labor markets are connected when they belong to the two-neighborhood of each other[1] (and call all the others unconnected labor markets). Heterogeneity in idiosyncratic workers' preferences regulates the sensitivity of labor supply to a location with respect to local wage differentials. The production side follows the framework proposed by Eaton and Kortum (2002), where I assume no internal transportation costs. In a closed economy version with one sector only, I separate theoretically the different components of the equilibrium adjustment of a location's wage to a shock in a direct effect, and indirect effects from connected and unconnected labor markets. I show, also with the aid of simulations, that the framework is already general enough to deal with the transmission of other types shocks, like immigration or technological change. I then extend the model to many tradeable sectors, one non-tradeable sector whose output can be only consumed in the location where it is produced, and open this economy to trade with the rest of the world. While the set of active sectors is exogenously given, the strength of comparative advantage regulates the sensitivity of employment's size across sector-locations to prevailing wages across locations. I show that we can understand the impact of a reduction in trade frictions with a decomposition similar to the one described above, and emphasize the role of intersectoral transmission and trade rebalancing after the shock.

While interesting at a theoretical level, the equilibrium interaction between local labor markets may remain an unnecessary complication if it is not shown to be empirically relevant. I focus my attention on a world made up of US and a Rest of the World (RoW) country. The model is flexible enough to incorporate realistic

[1] When the notation is laid out below, a precise definition accounting for the possibility of one-way commuting only will be given.

geographic features such as the set of observed commuting ties among locations and the active sectors throughout the US economy. I estimate the labor supply side of the model with data on commuting patterns among 1,230 Public Use Microdata Areas (PUMAs), covering the entire continental US, and find that transportation infrastructure is very important in raising the elasticity of labor supply. I estimate labor demand parameters using PUMAs employment by location and industry and standard international trade data by sector, and find that the strength of comparative advantage is similar in range to parameters already commonly used in the international trade literature. The contribution of each component of the transmission mechanism can be then computed in principle for any counterfactual exercise.

I use the estimated parameters to perform two counterfactual exercises meant to illustrate some important points in our understanding of the geographic incidence of international integration. I first study the consequences of the elimination of trade frictions in a single manufacturing sector, "Computer and Electronic Products" (CEP). This sector is interesting as it has very low trade frictions in the data (the measured CIF/FOB ratio is about 2%), is not active in about one-fifth of PUMAs, and is quite concentrated in terms of employment. I find that eliminating these small trade frictions would produce an increase of 0.22% in the wage paid in the median PUMA, and that variation in the impact across PUMAs is about 40% of this median value. Also, the order of magnitude of the impact in locations *not* active in the sector is comparable to the one where CEP is active. Finally, a model where commuting possibilities are shut down by construction would tend to over-predict the change in wages where CEP is active, and under-predict these changes where CEP is absent.

In a second counterfactual exercise, I study the consequences across US PUMAs of a tripling in the manufacturing productivity of the rest of the world. This is an interesting exercise as growth in the productivity of developing countries in general, and of China in particular in the last two decades, have been a first-order feature of the international environment in which US operates. The model suggests that changes in wages paid to workers are on average 1%, while the impact on real wages on the median PUMA is a little more than one-fifth of it. Importantly, there is basically no relation in this exercise between changes in the wages of workers and changes in the wages of residents. This lack of correlation should suggest caution when drawing welfare implications from changes in observed workers' wages.

The geographic incidence of trade shocks is the subject of a recent new stream of research, of which this paper is part. Several studies (Hanson 2005 and Chiquiar 2008 for Mexico, Topalova 2005, 2010, Hasan, Mitra, and Ural, 2006 and Hasan, Mitra, Ranjan, and Ahsan 2012 for India, McLaren and Hakobyan 2010, and Autor Dorn and Hanson 2013 for US[2], McCaig 2011 for Vietnam, Kovak, 2013 for Brazil) have documented empirically the effect of international trade integration on within-country geographical units of varying size, by relating changes in exposure to international competition to changes in local outcomes such as the incidence of poverty, unemployment, wages paid to workers, and the take-up of federal benefits. The present paper contributes to this literature by providing a tractable theoretical framework which is nonetheless able to accommodate realistic geographic features of a country's labor market, its sectoral concentration and locations' specializations. The analysis shows the importance of interconnectedness of local labor markets via commuting flows and overlaps in sectoral specialization across locations, features which are mostly absent from this literature, both at theoretical and empirical level: I compute that, for the average location, 60% of the effect on wages of the

[2] Early work from Borjas and Ramey1995 across U.S. metropolitan areas is an important precursor in this literature.

small liberalization in the CEP sector comes from something occurring in connected labor markets, 23% because of trade rebalancing, 9% from unconnected labor markets, and only 6% from a direct effect of trade. Moreover, the model suggests that measures of exposure to trade based on the share of employment in sectors impacted by liberalization are not adequate to capture the consequences of trade in locations where these sectors are not active; indeed, the first counterfactual simulation shows that locations inactive in CEP - which would have zero exposure by construction - experience consequences similar to those active in the sector.

The literature has analyzed geographical units of varying extension, from relatively small Indian districts (e.g. Topalova 2005), through commuting zones (Autor, Dorn and Hanson 2013) and consistent PUMAs (McLaren and Hakobyan 2010) for US, to relatively large Mexican regions (Hanson 2005 and Chiquiar 2008). The analysis in this paper indicates that having larger units of analysis is not necessarily sufficient to exclude linkages between labor markets: the model shows that, everything else equal, small commuting flows are indeed those who carry the highest potential for transmission, as they raise elasticities of labor supply, and that no commuting flows between two areas are not sufficient for two labor markets to be unconnected. The existence of commuting ties between locations introduces a theoretical distinction between nominal wages of agents supplying work to a location, and real wages of residents living in a location. While the latter is arguably a better indicator of welfare, lack of data on the price of non-tradeable goods (at the empirical level) and of modeling of commuting ties (at theoretical level) often implies a focus of the attention on the former. The second counterfactual exercise shows that, for a relevant empirical case, there may be an only very weak relation between changes in wages of workers, and changes in real wages of residents. This lack of correlation suggests caution in interpreting changes in wages of workers from a welfare perspective. The first counterfactual exercise also indicates that an approach that ignores commuting ties would tend to over-estimate the impact of the trade shock in locations very specialized in the liberalized sector (and vice-versa for those not specialized in it).

The literature on labor mobility, including for example Topel (1986), Blanchard and Katz (1992) Glaeser and Gyourko, (2005), Kennan and Walker (2011) and Notowidigdo (2013), typically argues that internal migration of labor is slow and incomplete. Autor, Dorn and Hanson (2013) show that within the two decades time frame they analyze there is only limited internal migration of workers across areas. The results in the present paper are based on a medium-to-long run perspective, where the distribution of employment across locations and industries is allowed to change, trade is allowed to rebalance, while the distribution of residents, commuting possibilities, and industries' location is kept fixed. The assumptions on labor mobility in this paper fall then mid-way between immobile labor across locations (e.g., Kovak 2013) and perfectly mobile labor across locations (e.g., Caliendo, Parro, Rossi-Hansberg and Sartre 2013).[3]

This paper is also similar in spirit to Caliendo et al. 2013, who study the effects of changes in technology in a state-sector on all other states in US, assuming perfect labor mobility and an explicit role for input-output tables and internal transportation costs. Here, I ignore input-output relations across industries, but allow for commuting and an open economy environment.[4]

Other studies offer structural models of labor markets, and characterize its dynamic adjustment to changes

[3] Kovak (2013) is a specific factor model where workers are specific to locations, while in Caliendo et. al (2013), workers are not attached to any geographical unit. The present paper could be viewed as a case of the specific factor model where labor is specific to a group of locations, and different location can use different, but overlapping, groups of factors.

[4] Other papers highlighting the importance of input-output matrices in open economy are Caliendo and Parro (2013), and Ossa (2013).

in the international environment. Contributions by Artuc, Chaudhuri and McLaren (2010), Artuç and McLaren (2012), Dix-Carneiro (2013), and Cosar (2013) show how to study transitional dynamics of trade shocks, integrating to varying degrees industry-specific and occupational specific-labor mobility costs. This paper assumes frictionless job mobility across sectors and locations (up to individual heterogeneity), no internal migration, and only compares steady states, but can focus sharply on the consequences of commuting and intersectoral concentration[5]. This focus allows us to study local mechanisms of transmission, and to depart from the common small open economy assumption to highlight the role of rebalancing of trade on the equilibrium change in wages.

This paper delivers a gravity structure for commuting flows similar to Ahlfeldt, Redding, Sturm and Wolf (2012), who develop a quantitative theoretical model of city structure and use exogenous variation induced by the rise and fall of Berlin's Wall to identify the role of agglomeration and dispersion forces and fundamentals in location choices. While our data is available at a more aggregated level, I show that the observed network of commuting ties can have deep implications on (and promising applications for) issues at lower spatial frequencies as well, like the impact of immigration, technology or trade shocks in general equilibrium analysis.

The paper is also related to a recent stream of literature (Allen and Arkolakis (2013), Caliendo et al (2013), Cosar and Fajgelbaum (2013), Donaldson (forthcoming), Donaldson and Atkin (2013), Donaldson and Hornbeck (2013)) that studies various aspects of the equilibrium interactions among within-country geographical units.

The rest of the paper is organized as follows. Section 2 sets up the model in closed economy, developing expressions for the elasticity of labor supply and demand to wages in different locations and describing the equilibrium. I use this simplified, one-sector framework in Section 3 to isolate theoretically the role of adjustment in commuting flows to the transmission of shocks and use simulations in Section 4 to illustrate these effects. I complete the model in Section 5 introducing location specialization in different industries and trade to emphasize the role of multiple sectors and trade balance in the geographic transmission of trade shocks. After describing some of the main characteristics of the data in Section 6, I show how to identify and estimate the main components of the model in Section 7. Section 8 uses these estimates to illustrate the transmission of the elimination of trade frictions in one sector on a particular geographical area of the US. Section 9 concludes.

2 Setup in Closed Economy

The economy is comprised by a set $\mathcal{R}$ of locations. Each location $r \in \mathcal{R}$ is inhabited by an exogenous number of H_r agents; total population in the economy is $H = \sum_{r \in \mathcal{R}} H_r$. These agents choose a job location and the expenditure on a continuum of varieties of unit length produced around the country.

Each worker provides 1 efficiency unit of labor. An individual living in $r \in \mathcal{R}$ can only have jobs in (i.e., commute to) a set of neighboring locations $\mathcal{J}(r) \subset \mathcal{R}$ which are close enough. We do not impose a priori restrictions on the sets $\{\mathcal{J}(r)\}_{r \in \mathcal{R}}$, as the structure of these sets can be reconstructed from the data on commuting patterns by identifying the origin-destinations pairs where at least one commuter is observed[6]. Since only a subset of locations is in the choice set of agents residing in a given r, and in general $\mathcal{J}(r) \neq \mathcal{J}(r')$

[5] I show in a separate Appendix that the framework is amenable to incorporate the choice of residence location in a fully dynamic, forward-looking setting along the lines of Artuc, Chaudhuri and McLaren (2010).

[6] In particular, for two locations r and j, we will say that $j \in \mathcal{J}(r)$ if we see at least one worker going from r to j. If the data never shows such a commuting, we infer that this pattern is not feasible, and we do not consider it in any counterfactual analysis. The collection of sets $\{\mathcal{J}(r)\}_{r \in \mathcal{R}}$ will be taken as a technological constraint.

if $r' \neq r$, the local labor supply becomes non-trivial. We assume that workers differ from each other in their idiosyncratic valuations of jobs in different locations: each worker living in r is characterized by a random vector $\left\{\chi_j\right\}_{j\in\mathcal{J}(r)}$ with $F_r(\chi) = \exp\left\{-e^{-\chi/\mu_r - \tilde{\gamma}}\right\}$, i.i.d across agents and locations, where $\mu_r > 0$ and $\tilde{\gamma}$ is the Euler's constant[7]. We assume the utility of choice j for an agent $\left\{\chi_j\right\}_{j\in\mathcal{J}(r)}$ to be

$$\ln w_j/\left(p_r d_{rj}\right) + \chi_j$$

where w_j is the prevailing wage paid in location j (which equates consumption of this agent[8]), p_r is the price index prevailing in location r, and $d_{rj} \geq 1$ is the reduction in utility in terms coming from a two-way commuting from r to j. A worker living in r takes as given the set of commuting possibilities $\mathcal{J}(r)$, the disutility consequences of commuting and the wage profile $\left\{d_{rj}, w_j\right\}_{j\in\mathcal{J}(r)}$, and solves

$$U_r(\chi) \equiv \max_{j\in\mathcal{J}(r)} \ln w_j/\left(p_r d_{rj}\right) + \chi_j \tag{1}$$

2.1 Consumption, production, and goods' flows

The economy has one sector of goods which can be freely shipped across the country. Later, we will extend the model to include different tradeable sectors and services which can only be consumed in the location where they are produced.

The production side of this economy follows closely Eaton and Kortum (2002)[9]. Any given expenditure w is allocated across a continuum of varieties of unit length according to the CES utility function

$$\left[\int_0^1 Q(i)^{(\sigma-1)/\sigma}\, di\right]^{\sigma/(\sigma-1)}$$

with $\sigma > 1$. In each location $j \in \mathcal{R}$ production of any of these varieties can occur. The efficiency of these varieties is described by a Frechet distribution, $\Pr\left\{Z_j < z\right\} = \exp\left\{-T_j z^{-\theta}\right\}$, where $T_j > 0$ is the state of technology in location j and $\theta > 0$ is a dispersion parameter. Each variety is produced under constant returns to scale with labor only, and sold in a perfectly competitive market: hence, the fraction of varieties produced in location j with unit cost less than $\bar{p}$ is $1 - \exp\left\{-T_j\left(w_j/\bar{p}\right)^{-\theta}\right\}$. Since consumers in any location r buy a variety only from its minimum cost provider, and we assume no transportation costs of goods in the internal market, the distribution of goods' prices actually consumed in any location is given by $G(\bar{p}) = 1 - \exp\left\{\Phi\bar{p}^{\theta}\right\}$, with $\Phi = \sum_{j\in\mathcal{R}} T_j w_j^{-\theta}$. Hence,

$$\pi_j = \frac{T_j w_j^{-\theta}}{\Phi}$$

is the share of goods that any residence r buys from location j, and also the share of any residence r's total income going to varieties produced in j. The absence of internal transportation costs implies that a) if location j is the

[7] Hence, $E(\chi) = 0$ and $Var(\chi) = \pi^2\mu_r^2/6$. With homogeneous locations, one would ideally restrict all μ_r to be the same. However, since in the data geographic areas have heterogeneous size and different degrees of transportation infrastructure, we will allow μ_r to be different.

[8] The allocation of consumption across different varieties is described in the next subsection.

[9] Other work adapting the Eaton Kortum (2002) framework to internal trade include Donaldson (forthcoming), Donaldson and Hornbeck (2013), Caliendo, Parro, Rossi-Hansberg and Sartre (2013).

minimum cost producer of a variety for a given destination, then it sells this variety everywhere in the national economy; and that b) the price index for the goods $p = \gamma \Phi^{-1/\theta} = p_r \; \forall r \in \mathcal{R}$, where $\gamma \equiv \left[\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{1/(1-\sigma)}$, is the same in all residence locations. While the former feature will not be modified, the latter will change once we introduce non-tradeable services in the economy.

2.2 Workers' flows and local labor supply

Given our assumptions on idiosyncratic preferences across locations and the utility maximization in (1), the fraction of individuals living in r and commuting to j is

$$m_{rj} = \frac{(w_j/d_{rj})^{1/\mu_r}}{\sum_{j' \in \mathcal{J}(r)} (w_{j'}/d_{rj'})^{1/\mu_r}} \tag{2}$$

Everything else equal, the fraction of residents of r commuting to j increases when fewer alternative options are available to residents of r (i.e., the lower the cardinality of $\mathcal{J}(r)$), and when the commuting costs to j are lower, or the commuting costs to alternative destinations $j' \neq j$ are larger. Note that the fraction of commuters from r to j is a function of all the wages $\{w_j\}_{j \in \mathcal{J}(r)}$ paid in all feasible destinations.

Since the total commuting flow from r to j is $H_r m_{rj}$, the local labor supply to any market j is given by

$$L(j) = \sum_{r: j \in \mathcal{J}(r)} H_r m_{rj} = \sum_{r: j \in \mathcal{J}(r)} H_r \frac{(w_j/d_{rj})^{1/\mu_r}}{\sum_{j' \in \mathcal{J}(r)} (w_{j'}/d_{rj'})^{1/\mu_r}} \tag{3}$$

The local labor supply to market j is then a function of $\cup_{r: j \in \mathcal{J}(r)} \{w_j\}_{j \in \mathcal{J}(r)}$, the union, over all residences r that can send workers to j, of all the wages that residents of r consider when evaluating j. Hence, this set includes also wages paid in locations that *do not send workers to j directly*, but are in competition with j for workers living in r.[10]

One key element in the transmission of shocks through the territory is the elasticity of labor supply to changes in wages. Consider for example what would happen following a negative demand shock to products manufactured in j. Suppose first commuting costs are prohibitive to and from j: the labor supply to j is given by all and only those living in j. Then, no worker would have the possibility of commuting outside, labor supply around j would not change, and - excluding general equilibrium effect - the shock would have to be completely absorbed by a fall in w_j, with no changes in equilibrium wages around j. Suppose instead commuting outside j is very easy: then a negative demand shock in j would induce people indifferent to working in j and somewhere else to switch away from j; local labor supply in neighboring locations would increase, and wages there decrease: the shock would be transmitted outside j, even if products manufactured outside it were not initially hit by the change in demand.

To understand the response of labor supply to j to changes in a generic wage $w_{j'}$, we need to first study the

[10] For example, in Figure 1 below (Section 4), location 3 cannot send workers to location 1 directly; nonetheless, wages in location 3 do affect labor supply in location 1 directly: if w_3 goes up, some of the residents in 2 who work in 1 will now switch and work in location 3 instead.

response of the fraction of commuters from r to j. From (2), the elasticity of m_{rj} to $w_{j'}$ is

$$\varepsilon\left(m_{rj}, w_{j'}\right) \equiv \frac{w_{j'}}{m_{rj}} \frac{\partial m_{rj}}{\partial w_{j'}} = \begin{cases} \left(1 - m_{rj}\right) / \mu_r & if\ j' = j \\ -m_{rj'}/\mu_r & if\ j' \in \mathcal{J}\left(r\right), j' \neq j \\ 0 & if\ j' \notin \mathcal{J}\left(r\right) \end{cases} \tag{4}$$

where $\varepsilon\left(a, b\right)$ will from now on indicate the elasticity of a to b. When the wage prevailing in j' increases, the fraction of commuters from r to j may be impacted in one of three ways.

If $j' = j$, the fraction of workers commuting to j increases as w_j grows, with elasticity $\left(1 - m_{rj}\right)/\mu_r$: when j is already very popular among these residents, most of the workers have already selected it, and only those for whom j is very unattractive remain to be convinced: the elasticity is then low. Given (2), m_{rj} is then very elastic to w_j if r has many alternative options, is very close to other areas, or is relatively far from j.

The fraction of workers going to any other destination $j' \neq j$ decreases as $w_{j'}$ grows, with elasticity $-m_{rj}/\mu_r$: when j is already very popular among these residents, also those for which j is relatively unattractive have selected it, and they only need a small increase in w'_j to be selected away. Hence, m_{rj} is very elastic to $w_{j'}$ when r has only a few other alternative options, is very far from other areas, or is relatively close to j.

Finally, there is obviously no direct effect on m_{rj} when $j' \notin \mathcal{J}(r)$, as $w_{j'}$ does not affect any choice margin.

A crucial role in these elasticities is played by μ_r. Formally, μ_r measures the variance in the individual heterogeneity χ, and thus regulates the sensitivity of commuting flows to differences in wages: when μ_r is very large, individual heterogeneity is all that matters and hence commuting flows do not respond to changes in wages; when, in the limit, $\mu_r \to 0$, all residents in r choose the highest-paying location only. In a world where the size and the transportation infrastructure change across residence locations, differences between μ_r will reflect how easy it is for residents to respond to changes in wages for given commuting penalties d_{rj}. As a preview, we will indeed find empirically that μ_r tends to grow with the measured area of residence locations, and decreases sharply with the total length of roads present in the area.

Using (4), the elasticity of the labor supply to location j with respect to the wage in j' is

$$\varepsilon\left(L\left(j\right), w_{j'}\right) \equiv \frac{w_{j'}}{L\left(j\right)} \frac{\partial L\left(j\right)}{\partial w_{j'}} = \begin{cases} \sum_{r:j\in\mathcal{J}(r)} \frac{H_r m_{rj}}{L(j)} \left(1 - m_{rj}\right)/\mu_r & if\ j' = j \\ -\sum_{\{r:j,j'\in\mathcal{J}(r)\}} \frac{H_r m_{rj}}{L(j)} m_{rj'}/\mu_r & if\ j' \in \mathcal{J}^2\left(r\right),\ j' \neq j \\ 0 & if\ j' \notin \mathcal{J}^2\left(r\right) \end{cases} \tag{5}$$

where $\mathcal{J}^2\left(j\right) \equiv \cup_{r:j\in\mathcal{J}(r)} \mathcal{J}\left(r\right)$ is the union, over all residences which can send workers to j, of all their possible destinations; in other words, $\mathcal{J}^2\left(j\right)$ is the set of all job locations whose prevailing wages can directly affect the labor supply in j.[11] We will say that j' is connected to j if $j' \in \mathcal{J}^2\left(j\right)$[12], and will call $\mathcal{J}^2\left(j\right)$ is the local labor market of j. The fact that local labor markets for different $j \in \mathcal{R}$ are overlapping allows shocks to unfold throughout the territory.

[11] The set $\mathcal{J}^2\left(j\right)$ is related to the concept of two-neighborhood of j, and it would indeed coincide with it if, in the network of commuting paths, edges between locations would be undirected. However, since we are in principle allowing $j \in \mathcal{J}\left(r\right)$ but not $r \in J\left(j\right)$, the equivalence doesn't necessarily carry through, as we may not be able to walk the network from j to j'.

[12] Note that the use of the term "connected" is somewhat non-standard here: we say that j' is connected to j even if there are no direct commuting flows between them.

Total labor supply to j responds to changes in $w_{j'}$ in one of three ways.

The elasticity of $L(j)$ to its own wage w_j is a weighted average of the elasticities of each commuting flow towards j, where the weights are given by the total contribution of residence r to location j's labor supply. Labor supply to j is very elastic if large contributors to j have many alternatives, or have low commuting costs to other locations (i.e., if large contributors have small commuting flows). Labor supply is very rigid if large contributors have just a few other alternative destinations, or those alternative destinations are very far.

The labor supply to j is also affected by wages prevailing in all locations connected to j: firms in these locations compete with firms in j for workers directly. For each r whose residents can choose between j and j', wage changes in any other $j' \in \mathcal{J}(r)$ can affect commuting flows towards j according to (4). The elasticity of labor supply to j with respect to changes in these $w_{j'}$ is more negative when the local labor market of j has many locations, and when j' is very popular among large contributors to j[13].

Finally, if j' is not connected to j, wage changes in j' will not affect labor supply in j directly.

Note that, up to μ_r, all the elasticities can be computed directly from data on commuting and location sizes.

To understand the response of equilibrium wages in any j, the second piece of information to consider is the elasticity of labor demand. This is the issue we tackle next.

2.3 Local labor demand

Let X denote the total national income in the economy,

$$X = \sum_{r \in \mathcal{R}} H_r \sum_{j \in \mathcal{J}(r)} m_{rj} w_j$$

From our analysis above, a share π_j of this expenditure is devoted to goods produced in location j, irrespective of the source of demand. Hence, the local labor demand in j is simply given by

$$D(j) = \pi_j X / w_j$$

It is easy to show that the elasticity of labor demand in j to a wage j' is given by

$$\varepsilon\left(D(j), w_{j'}\right) = \begin{cases} -\theta\left(1-\pi_j\right) + \frac{w_j}{X}\frac{\partial X}{\partial w_j} - 1 & if\ j' = j \\ \theta \pi_{j'} + \frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}} & if\ j' \neq j \end{cases}$$

When w_j increases, labor demand in location j changes for three reasons: 1) as w_j increases, labor demand decreases, everything else equal; 2) for given national income, an increase in w_j deteriorates the competitiveness of region j, π_j, and thus reduces labor demand in proportion to $\theta\left(1-\pi_j\right)$; 3) as w_j changes, commuting flows reallocate towards location j, and hence the national income changes for given relative wages; this third term is typically ambiguous. Note that in principle we cannot rule out that, in some regions, labor demand in j is increasing in its wage: the second effect may be large enough.

When $w_{j'}$ increases, on the other hand labor demand in location j changes for two reasons: 1) for given

[13] That is, when $m_{rj'}$ is high when $H_r m_{rj}/L(j)$ is also high.

national income, an increase in $w_{j'}$ improves the competitiveness of region j, π_j, and thus increases labor demand in proportion to $\theta\pi_{j'}$; 3) as $w_{j'}$ changes, commuting flows reallocate towards location j', and hence the national income changes for given relative wages (this third term is typically ambiguous). In Appendix B.1.1 we show that

$$\frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}} = \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} \iota_r \left(\frac{1+\mu_r}{\mu_r}\iota_{rj'} - m_{rj'}\frac{1}{\mu_r}\right) \tag{6}$$

where, denoting with $\tilde{w}_r \equiv \sum_{j\in\mathcal{J}(r)} m_{rj}w_j$ the average nominal wage of residents in r, the term $\iota_r \equiv H_r\tilde{w}_r/X$ represents the share of national income accruing to residents in location r, and $\iota_{rj} \equiv m_{rj}w_j/\tilde{w}_r$ is the share of income of residents in r coming from work performed in j. When this term is positive and large enough, labor demand in j becomes an increasing function of its own wage, i.e., $\varepsilon\left(D\left(j\right), w_j\right) > 0$.

2.4 Equilibrium

We are now in a position to write the system of equations that describe the general equilibrium. In an economy with R residential and work locations, we have R equations of the form

$$L\left(j\right) = D\left(j\right)$$

where the left-hand side describes the labor supply to location j given relative wages w_j and workers population H_j, and the right-hand side gives the labor requirements necessary to satisfy the expenditure on products from j, given relative wages w_j and technology profiles T_j. A number $R-1$ of these equations are independent, and determine the relative wages of each job location. These wages determine in turn the commuting flows m_{rj} and the flow of goods π_j. In the rest of the analysis in closed economy, we will set as normalization $p = \gamma\Phi^{-1/\theta} = 1$, i.e., $\sum_{j\in\mathcal{R}} T_j w_j^{-\theta} = \gamma^\theta$.

In Appendix B.1.2 we prove the following propositions:

Proposition 1 *An equilibrium w^* exists.*

Proposition 2 *Let w^* be an equilibrium; then this equilibrium is generically locally unique.*

We now turn to describe the impact of an exogenous change in a parameter H_k or T_k on an arbitrary - possibly the same - location j.

3 The structure of transmission

We start the exploration of the transmission of a shock in the closed economy model: this allows us to focus our attention on the effect of commuting flows when all locations are producing goods in the same industry; some simulations in the next section will illustrate these results. In the following section we will complete the model with many industries and trade, emphasizing intersectoral transmission.

We start from the analysis of the response to an increase in H_k, and move later to the analysis of a response of T_k.

3.1 A localized increase in the population

A change in the population of a region is relevant for foreign immigration (where the net flow is positive), or as a building block of internal migration (where the net flow across all regions is zero).

The impact of a change in H_k on the wage paid in location j can be decomposed in a direct effect, an indirect effect from connected labor markets, and an indirect effect from unconnected labor markets. The following proposition shows how:

Proposition 3 *The elasticity of the wage paid in any location j to the increase in the population in a location k is*

$$\begin{aligned}\varepsilon(w_j, H_k) \;=\;& \underbrace{\iota_k \frac{1-\iota_{kj}/\pi_j}{\varepsilon(L(j), w_j) - \varepsilon(D(j), w_j)}}_{\text{direct effect}} + \underbrace{\sum_{j' \in \mathcal{J}^2(j)\setminus\{j\}} \frac{\varepsilon(D(j), w_{j'}) - \varepsilon(L(j), w_{j'})}{\varepsilon(L(j), w_j) - \varepsilon(D(j), w_j)} \varepsilon(w_{j'}, H_k)}_{\text{indirect effect from connected labor markets}} \\ &+ \underbrace{\sum_{j' \in \mathcal{R}\setminus\mathcal{J}^2(j)} \frac{\varepsilon(D(j), w_{j'})}{\varepsilon(L(j), w_j) - \varepsilon(D(j), w_j)} \varepsilon(w_{j'}, H_k)}_{\text{indirect effects from unconnected labor markets}} \end{aligned} \tag{7}$$

where $\iota_k \equiv H_k \tilde{w}_k / X$ is the share of national income accruing to residents in location k (having an average wage of $\tilde{w}_k$ overall), $\iota_{kj} \equiv m_{kj} w_j / \tilde{w}_k$ is the share of income of residents in k coming from work performed in j, and the right-hand side is evaluated at the equilibrium w.

Proof. See Appendix B.1.3. ■

We describe the numerator of each of these three effects in what follows. The denominator, common across terms, will be described below.

The direct effect

The first term in the equation captures the net change in labor demand in j induced by an increase in H_k directly, i.e., not mediated through changes in other locations' wages.

As H_k increases, national income grows for given wages (as more workers live in k), and a share π_j of this increase is spent on products manufactured in j, raising labor demand; on the other hand, when $j \in \mathcal{J}(k)$, the increase in H_k also raises labor supply to j in proportion to m_{kj}. The net effect is captured by the sign of the numerator, which summarizes the imbalance, for each dollar of new income in k, between the fraction flowing out on products from j and the fraction flowing in as wages generated in j: if $\pi_j > \iota_{kj}$, an increase in H_k generates a net increase in labor demand, while if $\pi_j < \iota_{kj}$ there will be an excess labor labor supply. For example, if k workers cannot commute to j, an increase in H_k always increases labor demand through this channel; on the other hand, if a large fraction of residents in k tend to commute to j, an increase in H_k generates a net increase in labor supply.

Finally, if region k is economically small (low ι_k), marginal changes in H_k have an only mild direct impact on what happens in j.

The indirect effect from connected labor markets

An increase in H_k has an equilibrium impact on wages in locations $j' \in \mathcal{J}^2(j) \setminus \{j\}$, i.e. in locations connected to j: this change $\frac{H_k}{w_{j'}}\frac{dw_{j'}}{dH_k}$, which we take as given in this equation, will be determined by an expression similar to (7) applied to j'. A change in $w_{j'}$ transmits to j through changes in labor demand ($\varepsilon\left(D(j), w_{j'}\right)$) and labor supply ($\varepsilon\left(L(j), w_{j'}\right)$), creating a (generally ambiguous) labor imbalance in j. Suppose for example $w_{j'}$ grows. Labor demand in j changes for two reasons: 1) the competitiveness of products manufactured in j' deteriorates ($\pi_{j'}$ shrinks); 2) commuting flows are reshuffled towards j', thus changing the national income for given relative wages (this effect is ambiguous). Since $j' \in \mathcal{J}^2(j)$, the equilibrium change in $w_{j'}$ also affects labor supply to j, diverting workers away from it and towards j'. When the net imbalance is positive, the excess labor demand will contribute to an increase in w_j.

The indirect effect from unconnected labor markets

As H_k increases, wages in locations j' not connected to j will also change. Changes in wages in these locations will not divert commuting patterns away from j directly and influence j only via labor demand; as above, an increase in $w_{j'}$ will change labor demand in j via a reduction in j' competitiveness, and a change in national income X following a reshuffling of commuting flows.

To summarize, excess labor demand or supply in j can be created either directly, or indirectly, either via wage changes in j's local labor market (which affect demand and supply of labor in j), or outside its local labor market (which affect only demand). The net effect of these forces may induce either an excess labor demand or an excess labor supply: the denominator in each term regulates the response in the local wage needed to eliminate such disequilibrium. In the typical case of labor demand decreasing in its wage (and in general when demand is less elastic than supply), a larger sum of these elasticities will reduce the need for a change in wage, as more of the adjustment will occur through quantities.[14]

To evaluate the consequences of immigration in k for the average real wage $\tilde{w}_r$ of residents of a location r (recall that the price index is normalized to 1) we compute

$$\varepsilon\left(\tilde{w}_r, H_k\right) = \sum_{j \in \mathcal{J}(r)} \left[\frac{w_j}{\tilde{w}_r} + \frac{1}{\mu_r}\left(\frac{w_j}{\tilde{w}_r} - 1\right)\right] m_{rj} \varepsilon\left(w_j, H_k\right) \tag{8}$$

The elasticity of $\tilde{w}_r$ to H_k is a weighted sum of the elasticity of all the wages in $\mathcal{J}(r)$. Each weight reflect the importance of a destination (m_{rj}) corrected for the facts that, if j pays wages higher than the average, a change in its wage (for given commuting flows) and in the commuting flows (for given wages) is relatively more important. Note in fact that If $w_j = \tilde{w}_r \; \forall j \in \mathcal{J}(r)$ (i.e., there is no heterogeneity in wages across destinations), or if μ_r is very large (inelastic commuting flows), the change in commuting patterns plays no role ($\frac{1}{\mu_r}\left(\frac{w_j}{\tilde{w}_r} - 1\right) = 0 \; \forall j$); Moreover, note that wages for residents in a location r are affected by immigration in any other location, whether or not they are connected through labor markets.

The impact of immigration on the average wage in the economy $\bar{w} = \sum_{r \in \mathcal{R}} \frac{H_r}{H} \tilde{w}_r$ is ambiguous, and in

[14] In the case where $\varepsilon\left(L(j), w_j\right) - \varepsilon\left(D(j), w_j\right) < 0$, the labor demand is increasing (since the reshuffling of commuting flows (6) generates an increase in income large enough) and cuts the labor supply curve from above at the equilibrium. An excess labor demand is then eliminated by a *decrease* in wage which would reduce national income.

general it depends on where immigration occurs. In fact,

$$\frac{H_k}{\bar{w}}\frac{\partial \bar{w}}{\partial H_k}=\frac{H_k}{L}\left(\frac{\tilde{w}_k}{\bar{w}}-1\right)+\sum_{r\in\mathcal{R}}\iota_r\varepsilon\left(\tilde{w}_r,H_k\right) \tag{9}$$

If residents in location k increase, average income increases, other things equal, if those residents were earning above-average wage (first term); however, immigration also has the effect of changing wages paid in all locations in the economy, and through this channel, all the average wages paid to residents of all locations; the third term, a weighted average of those elasticities, capture this effect[15].

3.2 Response to an improvement in technology

An increase in T_k is an improvement in the distribution of the efficiency of ideas present in location k. Such an increase will improve the competitiveness of location k *vis-à-vis* all the other regions. In closed economy, such an increase may be driven by municipal policies aimed at improving the efficiency of firms, or by an improvement in the education of the local workforce. Following arguments similar to Proposition 3, it is easy to prove the following

Proposition 4 *The elasticity of the wage paid in any location j to the improvement in technology in a location k is*

$$\begin{aligned}\varepsilon\left(w_j,T_k\right) &= \underbrace{\frac{\varepsilon\left(\pi_j,T_k\right)}{\varepsilon\left(L\left(j\right),w_j\right)-\varepsilon\left(D\left(j\right),w_j\right)}}_{\text{direct effect}}+\underbrace{\sum_{j'\in\mathcal{J}^2(j)\backslash\{j\}}\frac{\varepsilon\left(L\left(j\right),w_{j'}\right)-\varepsilon\left(D\left(j\right),w_{j'}\right)}{\varepsilon\left(L\left(j\right),w_j\right)-\varepsilon\left(D\left(j\right),w_j\right)}\varepsilon\left(w_{j'},T_k\right)}_{\text{indirect effect from connected labor markets}} \\ &+\underbrace{\sum_{j'\in\mathcal{R}\backslash\mathcal{J}^2(j)}\frac{\varepsilon\left(D\left(j\right),w_{j'}\right)}{\varepsilon\left(L\left(j\right),w_j\right)-\varepsilon\left(D\left(j\right),w_j\right)}\varepsilon\left(w_{j'},T_k\right)}_{\text{indirect effects from unconnected labor markets}}\end{aligned} \tag{10}$$

where

$$\varepsilon\left(\pi_j,T_k\right)=\begin{cases}1-\pi_j & if\ k=j\\ -\pi_k & if\ k\neq j\end{cases}$$

Proof. Follows the same steps of the Proof of Proposition 3 in Appendix B.1.3. ■

The structure of the transmission is the same as for a change in H_k: an improvement in T_k has a direct effect on the labor market in j, and indirect effects through changes in wages of connected and unconnected labor markets. The impact of a direct effect in this case is very simple: if the improvement occurs in the technology of location j, this improvement in competitiveness translates into increases in labor demand in proportion to $1-\pi_j$. The direct effect then requires an increase in w_j (inversely related to the size of $\varepsilon\left(L\left(j\right),w_j\right)-\varepsilon\left(D\left(j\right),w_j\right)$) to restore the equilibrium; if the improvement occurs in the technology of another location, then labor demand

[15] In the appendix, we show that the weighted-average of the elasticities of $\tilde{w}_r$ can be equivalently understood as the impact of all the reshuffling of labor supplies to all job locations (through changes in commuting flows). This equivalence is a consequence of writing the average wage in terms of the job destinations.

decreases in proportion to π_k, and a decrease in the wage in j is required to restore equilibrium, other things equal.

3.3 Centrality

The above analysis expresses the elasticity of the wage of a location j to a change in parameter ζ_k, as a function of the elasticity of all the other relative wages, plus a direct effect, and can be written in compact form as

$$\boldsymbol{\varepsilon}(w;\zeta_k) = \boldsymbol{\xi} + \mathbf{A} \times \boldsymbol{\varepsilon}(w;\zeta_k)$$

where $\boldsymbol{\varepsilon}(w;\zeta_k)$ is an $R \times 1$ vector of elasticities with $j-th$ element $\varepsilon(w_j;\zeta_k)$, $\boldsymbol{\xi}$ is an $R \times 1$ vector of direct effects, $\mathbf{A}$ is a conformable square matrix containing the coefficients of all the indirect effects, and all terms are evaluated at the equilibrium wages. Note that this relation implies,

$$\boldsymbol{\varepsilon}(w;\zeta_k) = [I - \mathbf{A}]^{-1}\boldsymbol{\xi} \tag{11}$$

which is related to measures of centrality in network theory. In our economy, a location j has high centrality if its equilibrium elasticity is large: in this case, in fact, anything happening to the wage in j has large consequences for all other locations when appearing on the right-hand sides of eq. (7) or (10). Locations with a zero elasticity would absorb the shock and would not transmit it to other connected and unconnected labor markets.

4 Some simulation results

In this section we illustrate the mechanics of shock transmissions with two simple examples. In the first, we describe the impact of a marginal increase in the population of one location in a linear, three-locations country with fixed commuting costs, splitting the response to wages in their three components described in eq. (7). In this example, there are no unconnected labor markets, so the third term is muted by construction. In the second example, we consider at a linear, four-locations country and emphasize the contribution of unconnected labor markets in the transmission of immigration shocks. In all simulations we use $\theta = 8.28$ (following the preferred estimate in Eaton and Kortum (2002)), $\sigma = 3$, and $\mu = 0.2$.

4.1 A linear, three-locations country

We start by studying a three-locations, linear country, and the impact of an 1% increase in the country's population concentrated in one location. Such an experiment models, for example, the consequence of a small immigration episode in the economy.

A linear country is depicted in Figure 1. The set of residences is $\mathcal{R} = \{1, 2, 3\}$, there are no commuting costs to work in the residence locations, but $d_{rj} = 1.3$ if $r \neq j$ and r and j are neighbors. Arrows indicate the possible commuting patterns. We impose symmetry on everything else, i.e., $H_r = 1$, $T_r = 1$, $\forall r \in \mathcal{R}$.

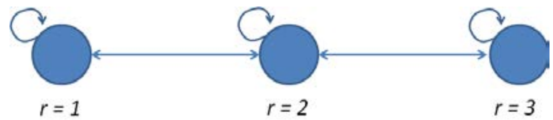


Figure 1: A 3 locations, linear country.

At these costs, the real wage paid in location 2 is $w_2 = 1.136$, while the real wage paid elsewhere is $w_1 = w_3 =$1.145. The real wage paid in location 2 is lower. The intuition is easier to grasp if we consider the case of no commuting costs. Were wages equal, residents in each location would split equally across each of their options: in this case, however, there would be an excess of labor supply in $j = 2$, as labor demand is the same everywhere. Hence, w_2 has to fall to eliminate the excess supply. Correspondingly, the matrix of commuting flows is given by

From / to	$j = 1$	$j = 2$	$j = 3$	Tot. Residents
$r = 1$	0.795	0.205	0	1
$r = 2$	0.180	0.640	0.180	1
$r = 3$	0	0.205	0.795	1
Tot. supply	0.975	1.050	0.975	3

Because of this lower wage, location 2 is more competitive and gets a larger share of expenditure of the total national income ($\pi_2 = 0.348$) compared to the other two locations ($\pi_1 = \pi_3 = 0.326$). Using (11), we can compute the equilibrium elasticities of a change in H_k on w_j, and using the decomposition in (7) we can separate the contribution of the direct and indirect effects. The following matrices report these results:

Total elasticity $\varepsilon(w_j, H_k)$

	H_1	H_2	H_3
w_1	-0.13	0.033	0.093
w_2	0.03	-0.063	0.03
w_3	0.093	0.033	-0.126

=

Direct effect

	H_1	H_2	H_3
w_1	-0.183	0.057	0.129
w_2	0.048	-0.096	0.048
w_3	0.129	0.057	-0.183

+

Ind. eff., connected l.m.

	H_1	H_2	H_3
w_1	0.057	-0.024	-0.033
w_2	-0.018	0.033	-0.018
w_3	-0.033	-0.024	0.057

As wages in location 3 can affect labor supply to location 1 (by diverting some of the residents in 2 away from it), there are no unconnected labor markets.

Let's focus our attention on the response to a change in H_1 (first column of each matrix). As the resident population increases, labor supply to 1 increases, other things equal, and the local wage is pushed down by 0.183%: this is the direct effect. However, wages in 2 and 3 will also respond positively in equilibrium: this changes labor demand in 1 as now location 2 and 3's products are less competitive and because commuting patterns change, affecting the national income; moreover, the increase in the wages divert labor supply away from 1. The net effects of these changes is to generate a (small) excess labor demand in 1, which increase the real wage paid in 1 by 0.057%, and hence the final effect is still negative: wages paid in 1 decrease by 0.13%.

Why are the wages in 2 and 3 increasing? In location 2, the increase in H_1 implies a larger national income (other things equal), a share π_2 of which falls on goods produced there; this induces an increase in labor demand, which is however partially compensated by a larger measure of commuters from 1, since total population there has increased: this direct effect induces an increase in real wage of 0.048%. Wages in 1 and 3 will move in opposite directions, generating contrasting forces on the labor market in 2 along the lines described above: the net effect is however to create an excess labor supply, which partially pushes down wages, by 0.018%. After the dust has settled, the net effect is still an increase in real wage of 0.03%. The adjustment in location 3 can be described similarly: notably, however, the net effect is larger (as to the same increase in national expenditure there is no direct increase in commuters supplying more labor), and the net effect on average wage (+0.093) is larger than the effect on the middle location.

Similar arguments can be made to trace the impact of a small change in H_2 (where now the effects on the neighboring locations is perfectly symmetric) or in H_3 (identical, *mutatis mutandis*, to the impact of H_1).

Using (8) and (9), we can also compute the effect of immigrations on wages by residence. In this example,

$\varepsilon(\tilde{w}_r, H_k)$	$k=1$	$k=2$	$k=3$
$r=1$	-0.096	0.014	0.082
$r=2$	0.013	-0.027	0.013
$r=3$	0.082	0.014	-0.096
$\varepsilon(\bar{w}, H_k)$	0.001	-0.002	0.001

Immigration in a location is always pushing down the wages of residents there, but the effects are quite heterogeneous: residents in the peripheral locations suffer a reduction of more than 3.5 times the one in central location: the reason is that location 2 is better connected in terms of job destinations, and can smooth this shock better. As discussed above, the effect on national average wage cannot be predicted just by knowing the magnitude of the increase in H_i, as it is also necessary to know *where* such an increase takes place. In this simulation, while the effects are very small due to symmetry, it still holds that immigration in residences 1 and 3 increase average wage (as their residents have an higher-than-average real wage), while immigration in the central location reduces it. Note also that average statistics are a very poor predictor of the distributions of effects across locations; when immigration occurs in location 1, for example, effects for different residents are between 13 and 96 times the average effect.

4.2 The role of unconnected labor markets

In this example we study a linear, four-locations country (see Figure 2): the set of residences is $\mathcal{R} = \{1, 2, 3, 4\}$ and we impose symmetry on everything ($H_r = 1$, $T_r = 1$, $\forall r \in \mathcal{R}$). We analyze the impact on w_1 of an immigration shock which increases the country's population by 1% and is concentrated in either of the four locations $k \in \mathcal{R}$. Having already understood how direct effects operate and the role of connected labor markets, we focus our attention on the contribution of unconnected labor markets as commuting costs progressively decrease from 2 to 1: in this example, we will study the role of location 4, which is the only one not part of location 1's local labor market.

Remember first that, as H_k increases, w_4 experiences an equilibrium change through the web of direct and indirect effects. The key to understanding the role of unconnected labor markets is to know a) whether the

Figure 2: A 4-locations, linear country.

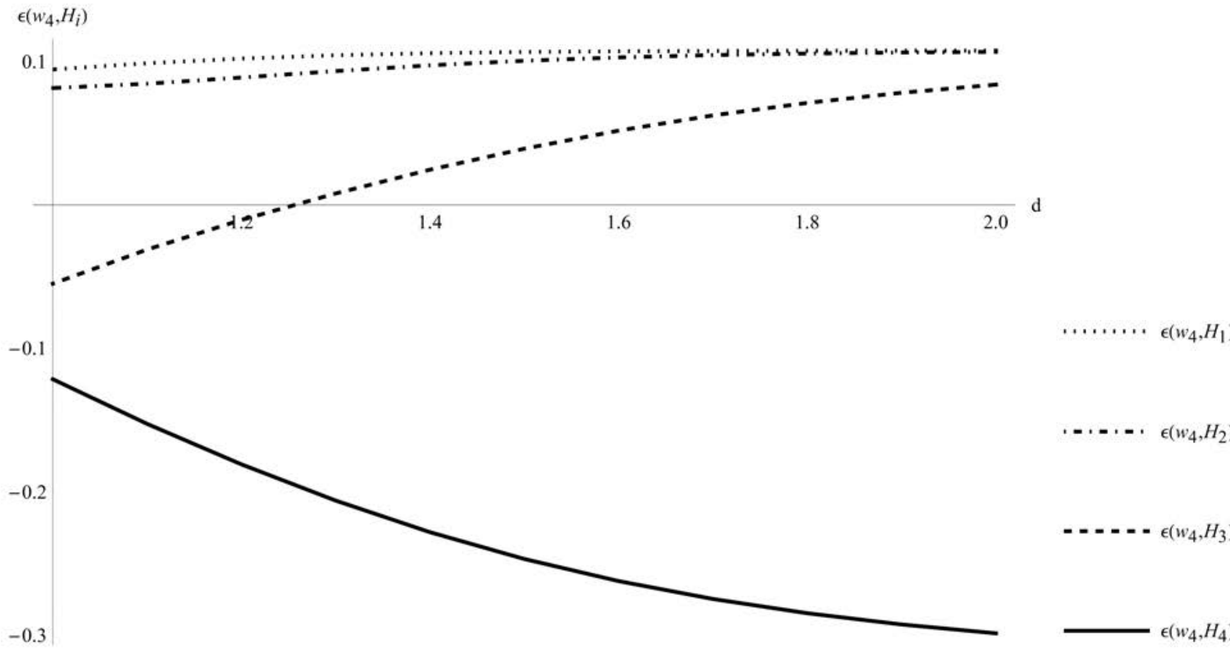


Figure 3: The response of w_4 to changes in H_k, for $k = 1, 2, 3, 4$.

equilibrium response of w_4 is upward or downward, and b) how will labor demand in 1 respond to that change.

Figure 3 shows the first point. The first thing to note is that, for fixed commuting costs, the farther is the shock from location 4, the more positive is the wage response: if immigration occurs far away, location 4 experiences only a limited increase in labor supply, but it does benefit from increases in labor demand. The second thing to note is that, as commuting costs become smaller, workers can more easily readjust via commuting to changes in relative wages, and hence all wages responses tend to be dampened: positive changes become less positive, negative changes become less negative.

A change in w_4 does not change labor supply to 1 directly by construction (as $1 \notin \mathcal{J}^2(4)$): it can only impact its labor demand. This impact occurs through two channels: first, the relative competitiveness of products in 1 will be higher if, other things equal, w_4 grows; second, as w_4 changes, the national income may change following a reshuffling of commuting flows: this changes expenditure on products everywhere, in general, and in location 1 in particular. Since whether or not w_4 grows depends - in part - on where the immigration occur, we need to evaluate the response to changes in each H_k separately.

Suppose first that immigration occurs in location 4. Figure 4 shows the net effect of an increase in H_4 to w_1, breaking down its different components; the red line, in particular, reports the indirect effect from location

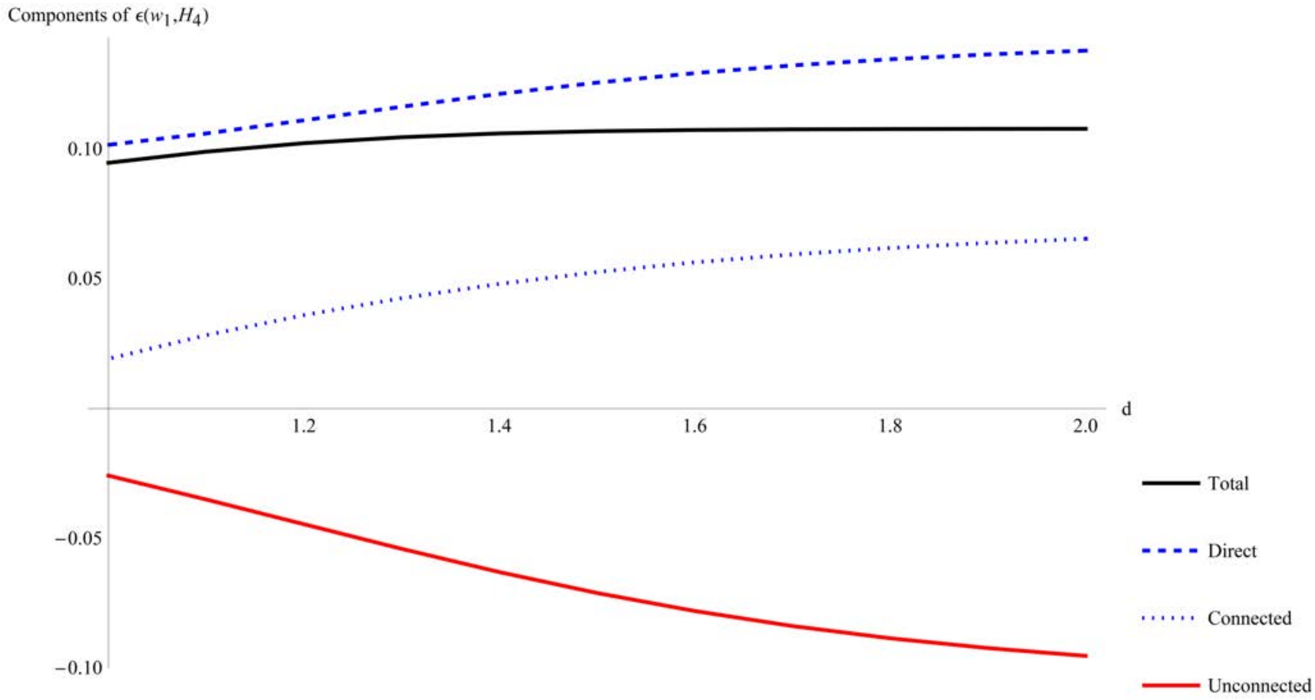


Figure 4: The response of w_1 to H_4, and its components.

4, the unconnected labor market.

In this case, w_4 will on balance decrease, as the increase in the local labor supply is only partially compensated by the indirect effects from other locations. Such a decrease implies that location 4 is now more competitive, which drives down labor demand in location 1. Moreover - especially for high commuting costs - national income decreases through this channel, since residents in location 4 can only partially move away from a region with declining wages, which further reduces labor demand in 1. What happens to wages in location 4 contributes negatively to the change in w_1: since the other two effects are positive, unconnected labor markets are limiting the rise of the wages paid in 1.

As the origin of the shock moves away from location 4 (and more towards location 1), w_4 responds more and more positively. Figure 5 shows the breakdown of direct and indirect effects of w_1 to H_1 (responses to H_3 and H_2 can be found in the Appendix, Figures 15 and 16). An increase in w_4 tends to raise labor demand in 1, other things equal, and hence the indirect effect from unconnected labor markets is to increase w_1. On the other hand, the direct effect of H_1 is heavily negative, and changes in connected labor markets are mildly positive. On balance, the presence of unconnected labor markets is slowing the fall in w_1.

The analysis so far has ignored the fact that sectors are, to varying degree, concentrated in space, and that locations are specialized in only a few sectors. Completing the model with these aspects and allowing for international trade will open up a further aspect of the transmission, and an additional transmission channel. We turn to these tasks next.

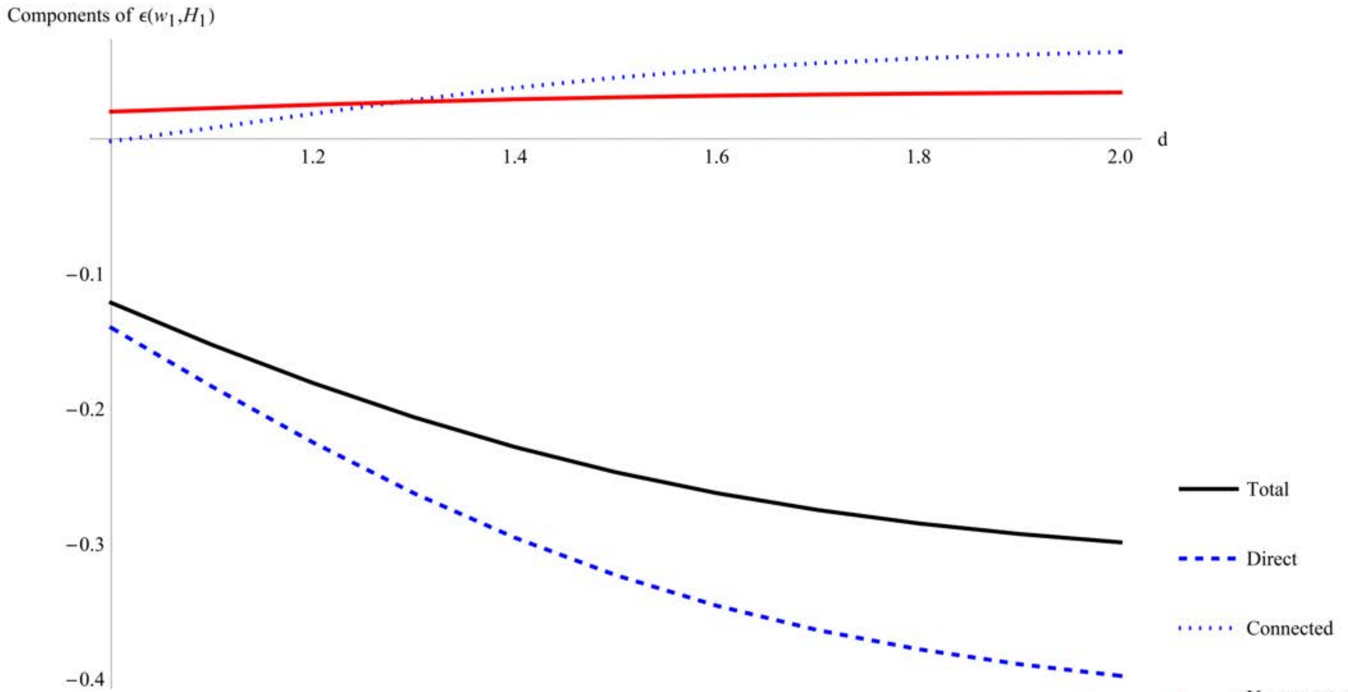


Figure 5: The response of w_1 to H_1, and its components.

5 Open Economy

I now introduce multiple sectors and international trade in differentiated varieties with an outside country. Variables referring to the outside country will have a star subscript. We denote with $\mathcal{S}$ the set of sectors with cardinality S. A subset $\mathcal{S}_M$ of these is of merchandise, tradeable both across locations at home and with the foreign economy; the remaining sector $\bar{s}$ is non-traded: residents of each location r can only consume services produced locally (of course, with all those working in r, irrespective of where they live). Let's denote with $\mathcal{S}_M(j)$ the set of tradeable sectors active in location j and let $\mathcal{S}(j) = \mathcal{S}_M(j) \cup \{\bar{s}\}$; the set of active sectors across locations will be taken as given. We extend the utility function to be a Cobb-Douglas aggregator of these goods, where the share of sector s is $\alpha^{(s)}$. Sectors are only different according to their technology parameter, $T_r^{(s)}$ at home and $T_*^{(s)}$ abroad, and they all use only labor. Letting w_* be the foreign country's wage and $\tau^{(s)}$ the trade costs, including tariffs, to ship a good from foreign to home (or vice-versa), the distribution of prices in the domestic country in sector s is given by $G_s(p) = 1 - \exp\left\{\Phi^{(s)} p^{\theta}\right\}$, where now

$$\Phi^{(s)} \equiv T_*^{(s)} \left(\tau^{(s)} w_*\right)^{-\theta} + \sum_{j \in \mathcal{R}} T_j^{(s)} w_j^{-\theta} \tag{12}$$

is also a function of trade costs. The fraction $\pi_{r*}^{(s)} = T_*^{(s)} \left(\tau^{(s)} w_*\right)^{-\theta} / \Phi^{(s)} \equiv \pi_*^{(s)}$ is the share of expenditure on sector s among residents in any location r on goods coming from abroad, and is decreasing in $\tau^{(s)}$ other things equal[16]: hence, $\pi_*^{(s)}\alpha^{(s)}X$ denotes total imports of the domestic country from abroad in sector s. Since the sector$-s$ price index in the domestic country is $\gamma\Phi^{(s)-1/\theta}$, the price index for the tradeable goods is $\prod_{s\in\mathcal{S}_M}\alpha_s^{\alpha_s}\left(\gamma\Phi^{(s)}\right)^{-a_s/\theta}$. The fraction $\pi_j^{(s)} = T_j^{(s)}w_j^{-\theta}/\Phi^{(s)}$ is still the share of expenditure on location j's products in sector s, bought by any home location.

Total expenditure of the foreign residents will be denoted as $X_* = w_*H_*$, where H_* is the measure of workers in the foreign country. The foreign country spends a fraction $\pi_{*j}^{(s)} = T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}/\Phi_*^{(s)}$ of its sector$-s$ expenditure $\alpha^{(s)}X_*$ on goods coming from location j in the home country, where

$$\Phi_*^{(s)} \equiv T_*^{(s)}w_*^{-\theta} + \sum_{j\in\mathcal{R}} T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta} \tag{13}$$

and hence a fraction $\pi_{**}^{(s)} = 1 - \sum_{j\in\mathcal{R}}\pi_{*j}^{(s)}$ on its own goods.

The labor supply to j and its elasticity to wages are always given by (3) and (5). A worker living in r and choosing location j is indifferent in which sector to work in, as they must all offer the same wage w_j. The distribution of workers across sectors in location j is then only driven by demand and expenditure shares.

To derive the local labor demand, we must now distinguish sectors and expenditure sources. For a tradeable sector s, $\pi_j^{(s)}\alpha^{(s)}X + \pi_{*j}^{(s)}\alpha^{(s)}X^*$ is the total expenditure on labor in j to serve domestic and foreign demand. The non-tradeable sector produces a homogeneous good under constant returns to scale and a productivity $T_j^{(\bar{s})}$; under perfect competition, equilibrium in the local market for such sector requires $\alpha^{(\bar{s})}\tilde{w}_jH_j/w_j$ units of labor in the sector, where $\tilde{w}_j$ is the average wage of residents in j.[17] Hence,

$$D(j) = \frac{1}{w_j}\left[\sum_{s\in S_M(j)}\left(\pi_j^{(s)}\alpha^{(s)}X + \pi_{*j}^{(s)}\alpha^{(s)}X_*\right) + \alpha^{(\bar{s})}\tilde{w}_jH_j\right] \tag{14}$$

describes labor demand in j.

The elasticity of labor demand to changes in any wage now encompasses several terms (see Appendix B.2.2 for details). The response of labor demand in j to its own wage, $\varepsilon(D(j), w_j)$ measures the consequences of i) the loss of competitiveness at home and abroad due to an increase in w_j in the sectors in which j is active, ii) the reduction in labor demand for given competitiveness because of a higher wage, and iii) the increase in the income of residents; in fact,

$$\begin{aligned}\varepsilon(D(j), w_j) &= \sum_{s\in S_M(j)}\left[\left(-\theta\left(1-\pi_j^{(s)}\right) + \frac{w_j}{X}\frac{\partial X}{\partial w_j}\right)\delta_j^{(s)} - \theta\left(1-\pi_{*j}^{(s)}\right)\delta_{*j}^{(s)}\right] \\ &\quad + \frac{w_j}{\tilde{w}_j}\frac{\partial\tilde{w}_j}{\partial w_j}\delta_j^{(\bar{s})} - 1\end{aligned} \tag{15}$$

[16] The fraction $\pi_*^{(s)}$ does not actually depend on r as we are assuming that $\tau^{(s)}$ is the same irrespective of the destination in the domestic country; this choice keeps the continuity with the assumptions on the domestic economy, where there are no within-country transportation costs.

[17] Equality between demand and supply requires $\alpha^{(\bar{s})}\left(\tilde{w}_jH_j + GH_j/H\right)/p_j^{(\bar{s})} = T_j^{(\bar{s})}L_j^{(\bar{s})}$, where $L^{(\bar{s})}$ is the labor force employed in $\bar{s}$. Since $p_j^{(\bar{s})} = w_j/T_j^{(\bar{s})}$ because of perfect competition, $\alpha^{(\bar{s})}\left(\tilde{w}_jH_j + GH_j/H\right)/w_j = L_j^{(\bar{s})}$. Note that in locations where wages w_j are high, the price of non-traded good $w_j/T_j^{(\bar{s})}$ is also high, consistent with a Balassa-Samuelson effect.

where $\delta_j^{(s)} \equiv \pi_j^{(s)}\alpha^{(s)}X/\left(w_j D\left(j\right)\right)$ and $\delta_{*j}^{(s)} \equiv \alpha^{(s)}\pi_{*j}^{(s)}X_*/\left(w_j D\left(j\right)\right)$ are the share of labor payments going to employees in the s sector for domestic and foreign sales, and $\delta_j^{(\bar{s})} \equiv \alpha^{(\bar{s})}\tilde{w}_j H_j/\left(D\left(j\right)w_j\right)$ is related to the share labor payments going to employees in the non-traded sector.

The response of labor demand in j to wages elsewhere, $w_{j'}$ is instead:

$$\varepsilon\left(D\left(j\right),w_{j'}\right)=\begin{cases} \sum_{s\in S_M(j)\cap S_M(j')}\theta\left(\pi_{j'}^{(s)}\delta_j^{(s)}+\pi_{*j'}^{(s)}\delta_{*j}^{(s)}\right)+\frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}}\delta_j^M+\frac{w_{j'}}{\tilde{w}_j}\frac{\partial\tilde{w}_j}{\partial w_{j'}}\delta_j^{\bar{s}} & if\ j'\neq j,\ j'\in\mathcal{J}\left(j\right)\\ \sum_{s\in S_M(j)\cap S_M(j')}\theta\left(\pi_{j'}^{(s)}\delta_j^{(s)}+\pi_{*j'}^{(s)}\delta_{*j}^{(s)}\right)+\frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}}\delta_j^M & if\ j'\neq j,\ j'\notin\mathcal{J}\left(j\right)\\ \sum_{s\in S_M(j)}\left[\delta_j^{(s)}\theta\pi_*^{(s)}+\left(1+\theta\pi_{**}^{(s)}\right)\delta_{*j}^{(s)}\right] & if\ j'=* \end{cases} \tag{16}$$

where $\delta_j^M \equiv \sum_{s\in S_M(j)}\delta_j^{(s)}$.

If j' is not a commuting destination ($j\neq j'$ and $j'\notin\mathcal{J}\left(j\right)$), producers in j in all sectors s where j and j' compete head-to-head have a relatively lower cost when $w_{j'}$ grows, thus raising labor demand. Moreover, a change in $w_{j'}$ has an impact on national income X (through reshuffling of commuting flows towards j', eq. (6)), thus changing labor demand in j for given competitiveness.

If $j'\in\mathcal{J}\left(j\right)$, labor demand in j also changes directly since the income received by residents in j varies, and this affects labor demand for non-tradeable goods.

When $j'=*$ we are evaluating the elasticity of labor demand in location j to changes in the foreign wage; an increase in w_* unambiguously raises $D\left(j\right)$ as j becomes more competitive with respect to foreign producers both at home and abroad, and total foreign expenditure X^* also increases for given competitiveness.

The equilibrium in open economy is a simple extension of the closed economy: we have $R-1$ equations of the form $L\left(j\right)=D\left(j\right)$, plus a trade balance condition of the form

$$\sum_{s\in\mathcal{S}_M}\pi_*^{(s)}\alpha^{(s)}X=\sum_{j\in R}\sum_{s\in\mathcal{S}_M(j)}\pi_{*j}^{(s)}\alpha^{(s)}X_* \tag{17}$$

Total imports of all sectors must be equal to total exports (i.e., the sum of sales abroad of all active sectors in j, for all j). Existence and local uniqueness are proven similarly to the closed economy case: the foreign market is just one additional labor market with no commuting possibilities.

Since each location has a different prevailing wage w_j, it also has a different price for the non-traded good, $T_j^{(\bar{s})}/w_j$. In what follows, we will normalize the price index of the tradeable sectors to 1, so that the price index for residents in r will simply be

$$p_r=\alpha_{\bar{a}}^{-\alpha_{\bar{s}}}\left(w_r/T_r^{(\bar{s})}\right)^{\alpha_{\bar{s}}}$$

5.1 Response to a change in trade costs

In this section we study the transmission mechanism of a small, symmetric reduction in one $\tau^{(s)}$ between the two countries. We first state the analog of Propositions 3 and 4, and then discuss how the elasticities of each term are modified in open economy. In the next section we will present some simulations which highlight the role of geographic concentration and specialization in the spreading of a trade shock.

Proposition 5 *The elasticity of the wage paid in any location j to a change in tariff in sector s, $\tau^{(s)}$, is*

$$\varepsilon\left(w_j,\tau^{(s)}\right) = \underbrace{\frac{\varepsilon\left(D(j),\tau^{(s)}\right)}{\varepsilon\left(L(j),w_j\right)-\varepsilon\left(D(j),w_j\right)}}_{\textit{direct effect}} + \underbrace{\sum_{j'\in\mathcal{J}^2(j)\backslash\{j\}}\frac{\varepsilon\left(D(j),w_{j'}\right)-\varepsilon\left(L(j),w_{j'}\right)}{\varepsilon\left(L(j),w_j\right)-\varepsilon\left(D(j),w_j\right)}\varepsilon\left(w_{j'},\tau^{(s)}\right)}_{\textit{indirect effect from connected labor markets}} +$$
$$+\underbrace{\sum_{j'\in\mathcal{R}\backslash\mathcal{J}^2(j)}\frac{\varepsilon\left(D(j),w_{j'}\right)}{\varepsilon\left(L(j),w_j\right)-\varepsilon\left(D(j),w_j\right)}\varepsilon\left(w_{j'},\tau^{(s)}\right)}_{\textit{indirect effect from unconnected labor markets}} +$$
$$+\underbrace{\frac{\varepsilon\left(D(j),w^*\right)}{\varepsilon\left(L(j),w_j\right)-\varepsilon\left(D(j),w_j\right)}\varepsilon\left(w_*,\tau^{(s)}\right)}_{\textit{indirect effect from trade balance}} \tag{18}$$

for a wage in a domestic economy, and

$$\frac{\tau^{(s)}}{w^*}\frac{dw_*}{d\tau^{(s)}} = \underbrace{\frac{\varepsilon\left(-NX(s),\tau^{(s)}\right)}{\sum_{s\in\mathcal{S}_M}\varepsilon\left(NX(s),w_*\right)}}_{\textit{direct effect}} + \underbrace{\frac{\sum_{j\in\mathcal{R}}\left(\sum_{s\in\mathcal{S}_M}\varepsilon\left(-NX(s),w_j\right)\frac{\tau^{(s)}}{w_j}\frac{dw_j}{d\tau^{(s)}}\right)}{\sum_{s\in\mathcal{S}_M}\varepsilon\left(NX(s),w_*\right)}}_{\textit{indirect effect}} \tag{19}$$

in the foreign economy. In these expressions,

$$\varepsilon\left(D(j),\tau^{(s)}\right) = \begin{cases}\theta\pi_*^{(s)}\delta_j^{(s)}-\theta\pi_{**}^{(s)}\delta_{*j}^{(s)} & if\ s\in\mathcal{S}(j)\\ 0 & if\ s\notin\mathcal{S}(j)\end{cases}, \tag{20}$$
$$\delta_j^{(s)} \equiv \pi_j^{(s)}\alpha^{(s)}X/\left(w_jD(j)\right)$$
$$\delta_{*j}^{(s)} \equiv \alpha^{(s)}\pi_{*j}^{(s)}X_*/\left(w_jD(j)\right)$$

and $\varepsilon(-NX(s),b)$ is the elasticity of net exports of sector s to b, for $b\in\left\{\tau^{(s)},w_*,w_j\right\}$

Proof. See Appendix B.2.3. ■

A change in tariffs has first of all a direct effect on labor demand in location j if this location is active in the sector, i.e, $s\in\mathcal{S}(j)$. A decrease in trade costs makes location j's producers less competitive at home but more competitive abroad, relative to foreign producers. In general, the net effect on labor demand depends on the relative strength of these two changes, weighted by the share of the labor employed for production in each destination.

Even if sector s is not active in location j, changes in $\tau^{(s)}$ can still have an impact on wages there. In the first place, for a connected labor market j', an increase in $w_{j'}$ will decrease labor supply in j (through (5)), thus inducing an upward pressure on w_j.

Furthermore, $w_{j'}$ can impact labor demand in j as well, whether or not these two labor markets are connected. Equation (16) shows, in fact, that a) when $w_{j'}$ grows, location j producers in all sectors which both j and j' are active will benefit from increased competitiveness; and b) that if j' is a commuting destination from j, labor demand in j also grows directly since the income received by residents in j changes, and this affect labor demand for non-tradeable goods. These effects come in addition to the usual change in national demand through reshuffling of commuting flows.

The equilibrium change in w_* increases labor demand in $D(j)$ since j becomes more competitive and national income abroad grows, other things equal. The response of w_* to changes in $\tau^{(s)}$ works through the trade balance condition (17), and is summarized in eq. (19), to which we now turn[18].

This equation describes - for given changes in all the wages in the home economy - the response of w^* to $\tau^{(s)}$ necessary to keep the trade balance in equilibrium.

The elasticity of w_* to $\tau^{(s)}$ has again a direct effect and an indirect effect. As $\tau^{(s)}$ changes, net imports of the home economy respond directly (for fixed wages): if net imports increase on net as $\tau^{(s)}$ decrease, w_* must also increase - making imports less competitive and exports easier - to restore trade balance; the magnitude of this change is regulated by the response of overall net exports to w_* at the denominator: a large, positive elasticity requires a smaller adjustment to w_*.

As $\tau^{(s)}$ decreases, wages in all locations j change through (18): as a consequence, net imports in each sector s also change: this is the indirect effect, and is a consequence of a non-trivial geography. In principle, net imports can increase or decrease through this channel: if they are on balance higher, then again w_* must increase to restore trade balance, and the adjustment is tempered by the sensitivity of net exports to w_* in the denominator.

As described in the closed economy, the denominator in all terms in (18) measures the sensitivity of the excess labor supply in j to changes in its own wage: the larger $\varepsilon(L(j), w_j) - \varepsilon(D(j), w_j)$, the smaller the adjustment in wages in j necessary to bring back the equilibrium in this local labor market. As argued above, the elasticity of labor demand now encompasses losses of competitiveness at home and abroad, the reduction in labor demand for given competitiveness, and the increase in the income of residents, all following an increase in w_j.

Locations whose wage is more elastic to $\tau^{(s)}$ are also the most central in transmitting shocks to neighboring regions.

As in the closed economy above, the impact of trade on the average wage of residents in a location j can easily be written as a weighted average of the elasticities of the contribution of each destination the average wage in the residence:

$$\frac{\tau^{(s)}}{\tilde{w}} \frac{\partial \tilde{w}_r}{\partial \tau^{(s)}} = \sum_{j \in \mathcal{J}(r)} \left[\frac{w_j}{\tilde{w}_r} + \frac{1}{\mu_r} \left(\frac{w_j}{\tilde{w}_r} - 1 \right) \right] m_{rj} \varepsilon \left(w_j, \tau^{(s)} \right)$$

Given the expression for the price index in r, the elasticity of the average real wage is then

$$\frac{\tau^{(s)}}{\tilde{w}} \frac{\partial \tilde{w}_r}{\partial \tau^{(s)}} - \alpha_{\bar{s}} \varepsilon \left(w_r, \tau^{(s)} \right)$$

5.2 Trade liberalization in concentrated sectors

In this subsection, we show by means of a simulation how trade liberalization in a sector not active in a location can nonetheless impact it. We simulate a linear, 4-locations country where 4 tradeable sectors are present. Each of the first three locations j is completely specialized in a sector $s = j$, while the fourth location is active in both sector 1 and 4 (see Figure 6). We study the impact on wages in location 1 of a trade liberalization in

[18] A more detailed expression for each of these terms can be found in the Appendix.

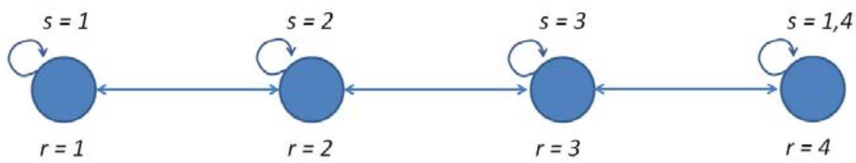


Figure 6: A 4 locations, linear country with concentrated sectors and specialized locations.

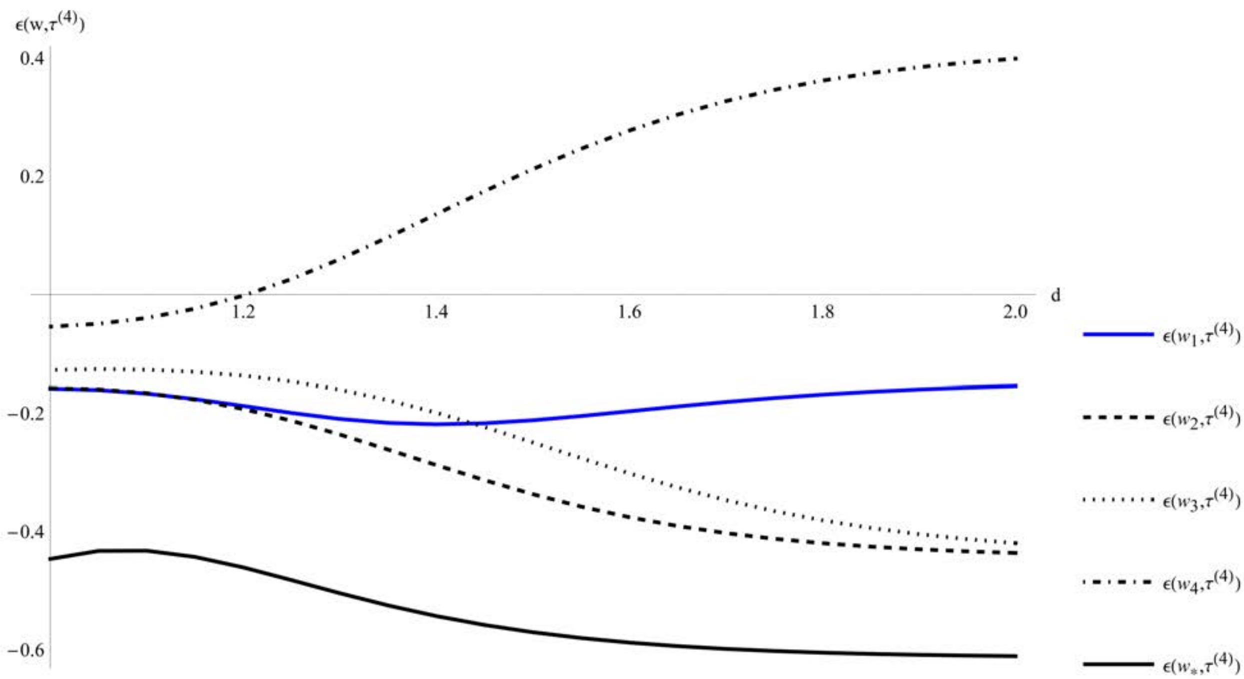


Figure 7: The response of all wages to increases in $\tau^{(4)}$.

sector 4, again as commuting among job locations moves from being difficult to being free. Note that location 1 is neither active in sector 4, nor connected to location 4 through local labor supply shifts. The economy is such that $H_r = 1\ \forall r \in \mathcal{R}$, $H_* = 4$, $T_j^{(s)} = 1$ in active sectors and 0 otherwise, $T_*^{(s)} = 1\ \forall s \in S_M$, $T_j^{s_{\bar{M}}} = 1$ everywhere, and each country spends 60% of income on non-tradeable, and the remaining 40% is equally split across each of the tradeable ones. Trade barriers are $\tau^{(s)} = 1.3$, and we will study the impact of a 5 percentage point reductions in $\tau^{(4)}$ on w_1.

The first figure (Figure 7) shows the equilibrium response of all wages to changes in $\tau^{(4)}$. A reduction of 5 percentage points in this parameter implies an equilibrium increase of 0.15-0.2 percent in the wage paid in location 1 despite the absence of commuting relations and sector 4 not being active in 1. To understand how the reduction in $\tau^{(4)}$ is transmitted to 1, take as given for now the fact that, in equilibrium, wages in 2, in 3, and abroad will increase, while the wage in location 4 decreases for most of the range of commuting costs.

The decomposition of the different effects is shown in Figure 8. As w_2 and w_3 grow, labor supply to 1 decreases, everything else equal, since more people commute from 1 to 2, and some of the commuters from 2 to 1 are diverted away, towards 3 (eq. (5)). Moreover, despite 1 does not compete head-to-head in any sector with 2 and 3, changes in w_2 and w_3 reshuffle commuting flow through (6) and the change in the average wage of residents in 1 affects labor demand in 1. The net direct effect of these connected labor markets is to increase the equilibrium wage between 0.08 and 0.12 percent. Note that as commuting costs decrease, this effect is stronger as stronger is the diversion of labor supply away from 1. Figures 17 and 18 in the Appendix show that w_2 and w_3 grow mostly in response to the rebalancing of trade across all sectors.

The wage in location 4 will in equilibrium *decrease* for a large range of high commuting costs. As this happens, producers in location 4 become more competitive both in sector 4 (the one hit by liberalization) and in sector 1: producers in location 1, which is specialized in sector 1, face now a tougher competition and decrease their labor demand, which dampens the increase in w_1 that would have been otherwise been observed (hence, the effect from unconnected labor markets is to reduce the wage as $\tau^{(4)}$ decreases). The decrease in w_4 is mostly driven by the direct effect (20) (see Figure 19 in Appendix): as $\tau^{(4)}$ goes down, producers in 4 become less competitive at home and more competitive abroad *vis-à-vis* foreign producers; the first effect prevail, labor demand falls and wages adjust downward. Note that as commuting costs decrease, the reduction in labor demand can be better accommodated by a shift of labor supply away from 1, and the direct effect is dampened. When commuting becomes very easy, w_4 also increases, the unconnected labor market *reinforces* the upward tendency in labor demand and w_1 goes up further.

Finally, as w_* increases, foreign producers become less competitive both in the domestic and the foreign market in sector 1, among others; everything else equal, this raises the demand for sector 1 products coming from location 1, and hence drives labor demand up. The reason w_* goes up is twofold, and shown in Figure 20. First, as $\tau^{(4)}$ decreases, net exports of sector 4 goods from the foreign country grow (the direct effect in eq. (19)); second, all wages in the home economy respond to changes in $\tau^{(4)}$, and this also, on balance, increase net exports from abroad (the indirect effect in eq. (19)). For trade to be balanced, this trade surplus needs to be eliminated via an increase in w_*. Note that, as commuting becomes easier, the direct effect is dampened: since more of the adjustment can occur on labor flows rather than wages, wages change less (Figure (7)) and net imports from foreign react less to changes in τ.

6 Data

In this section we turn to the description of data used, emphasizing some of their basic features. We start by describing data on commuting patterns, and then turn to employment, wages, and sectoral composition. Data on expenditure shares across sectors, absorption, and trade frictions will be referred to when explaining the calibration of trade costs and expenditure shares in the next section.

6.1 Commuting

I construct commuting patterns using information in the US Census of 2000, 5% sample and the American Community Survey (ACS) from 2005 to 2007. Data on commuting patterns are not available in the ACS before 2005. I stop in 2007 because ACS stops reporting data on weeks of work on a continuous basis (an information used when computing hourly wages) and only uses intervals, and to avoid contaminating the analysis with the impact of the financial crisis. The universe comprises all employed people who do not live or work in Alaska,

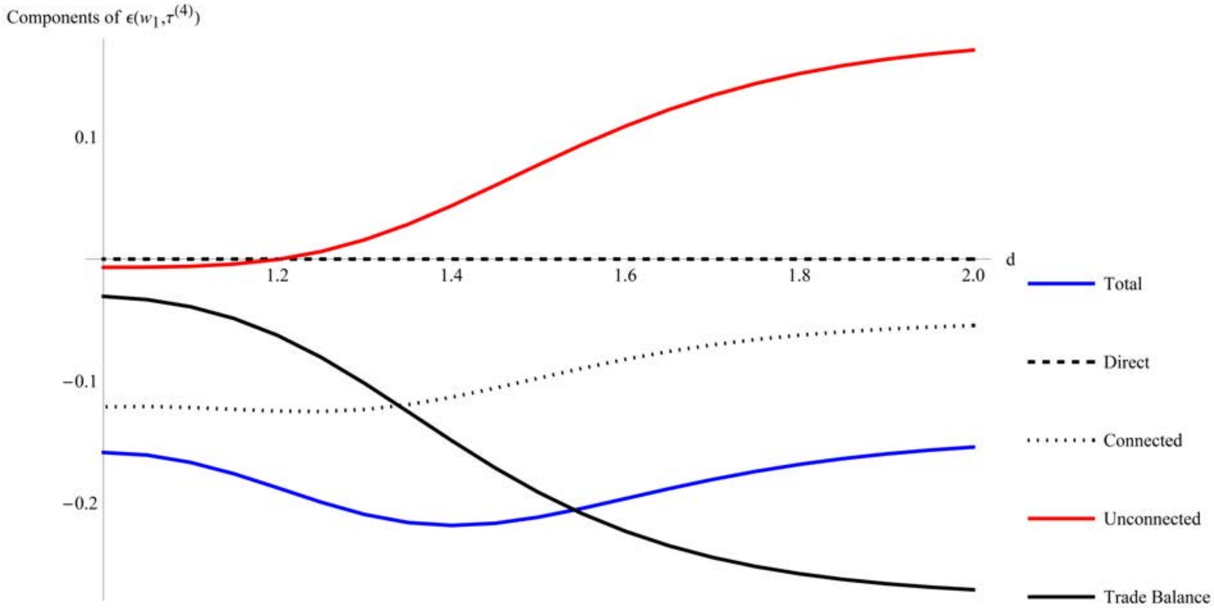


Figure 8: The equilibrium response of w_1 to $\tau^{(4)}$, and its components.

Hawaii, or Puerto Rico, or abroad, do not work in the Armed Forces, and have valid information on the place of work.

A commuting pattern in the data is a reported place of residence and place of work. Since the model does not allow for individual level variation in hours supplied, I choose the empirical counterpart of m_{rj} as the fraction of hours of employment from workers living in r which are supplied to j (and similarly, I identify employment in a location j with the total hours of work supplied in a year to j from residents anywhere). Geographic information on the place of residence and of work comes in PUMAs (Public Use Microdata areas). PUMAs are statistical geographical areas that contain a population of at least 100,000 persons in the 2000 Census, and provide the smallest geographical area available for public use in the Census (5% sample) and ACS (from the year 2005 onward). PUMAs of residence and of work use in most cases the same coding; occasionally, however, different PUMAs of work are combined into a single larger PUMA of residence: in these cases, I aggregate the PUMAs of work as well to obtain a consistent sample of 1,230 areas where people can live or work.[19]

Table 1 shows the distribution of number of destinations reached by workers resident in each PUMA, by year (i.e., statistics on the out-degree distribution of the commuting network). The median PUMA has workers commuting to about 22 destinations (when using ACS data, 2005-2007), and 47 destinations, when using Census data. There is ample heterogeneity across PUMAs in the out-degree of each. The large difference between Census and ACS statistics are due to the fact that Census samples many more individuals than ACS, and hence it is more likely to find individuals commuting to rare destinations[20]. In any case, estimates of the

[19] Due to Hurricane Kathrina, some PUMAs in Louisiana have been aggregated in 2007 as they would have not met the population requirements otherwise. I aggregate them in the previous years as well for consistency.

[20] A number of robustness checks have been performed to make sure this is a reasonable explanation. While gas prices increased almost 4 times from 2000 to 2005, I found no changes in commuting times, commuting modes, or in commuting time by mode. The

parameters of the elasticity of labor supply in the next section are performed using all the data, and then 2 subsamples (only year 2000, only 2005-2007, with and without time dummies) and there is no relevant change in our parameters of interest.

Table 1: Number of destinations by PUMA, unweighted

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	17	30	37	47	62	79	257	52.84	1,230
2005	5	13	17	22	31	43	120	25.87	1,230
2006	6	13	17	22	31	44	121	26.16	1,230
2007	5	14	18	23	32	45	120	26.91	1,230
ALL	5	14	19	27	41	58	257	32.94	4,920

This table reports selected percentiles in the out-degree distribution of commuting patterns, i.e. the number of PUMAs of work chosen by at least one worker from any PUMA of residence. The first row uses Census 2000, the following three ACS in the reported years, and the last aggregates all years. Each PUMA of residence receives weight 1.

A concern we may have is that the wide out-degree distribution is driven only by some small PUMAs. In Table 14, Appendix A.1, I replicate Table 1 weighting each observation with the total hours of work supplied by residents in each PUMA, and find that the distribution has actually a right-ward shift. I conclude that, despite being sizeable in terms of population, PUMA areas leave room for commuting patterns to emerge in the data.

Even if the number of job locations typically reached is well above 1, it may still be that the fraction of labor supplied represented by those hours commuting outside is small enough to make any analysis economically insignificant. To address this concern, I compute for each PUMA of residence the maximum fraction of hours across all its observed destinations, $\max_j m_{rj}$: ideally, we would like a distribution with a non-trivial left tail, i.e., that PUMAs of residence have most popular destinations which do not account for the almost totality of hours supplied[21]. Table 2 shows that this is indeed the case: for half of the PUMAs, more than one-third of hours supplied commute outside.

A crucial part of the transmission mechanism relies on the distance between two PUMAs: the disutility of commuting from r to j is a function, among other things, of the distance between them. I approximate the distance between PUMAs r and j computing the distance between their centroids[22], as the data does not provide individual distances traveled for work reason. Summary statistics on these distances across all workers are presented in Table 3.

drop occurs across all States. I find that 99% of workers in 2000 commute to destinations which are also present in ACS: hence, it is only a small fraction of individuals which accounts for a halving in the number of destinations. These considerations all support the conjecture that the difference is just due to sampling.

[21] In a strong majority of cases, the most popular PUMA of work is the PUMA of residence. In 8% of the PUMA of residence-year observations (392 out of 4920) this does not happen: these PUMA-years are on average 3.5 smaller in terms of size, have a 1.5 denser transportation network (total length of major and minor highways, major roads, and railroads divided by PUMA area) and are about twice as dense in terms of hours of labor supplied by residents, or resident population at or above 16 y/o, per square mile of PUMA.

[22] I compute the distance using the Haversine formula; the distance between r and j is

$$2 \times R \times \arcsin\left(\sqrt{\sin^2\left(\frac{lat_j - lat_r}{2}\right) + \cos\left(lat_r\right)\cos\left(lat_r\right)\sin^2\left(\frac{lon_j - lon_r}{2}\right)}\right)$$

where lat and lon represents latitude and longitude in radians of the two centroids. Centroids of each PUMA have been computed using standard spatial analysis software and requiring that the centroid falls within the PUMA itself.

Table 2: Distribution of $\max_j m_{rj}$ across all residences

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	0.16	0.34	0.49	0.67	0.82	1	1	0.64	1,230
2005	0.16	0.35	0.48	0.66	0.81	1	1	0.64	1,230
2006	0.18	0.35	0.48	0.66	0.80	1	1	0.64	1,230
2007	0.16	0.36	0.48	0.65	0.81	1	1	0.64	1,230
ALL	0.16	0.35	0.48	0.66	0.81	1	1	0.64	4,920

This table reports selected percentiles in the distribution of the fraction of hours commuting to the most popular destination. Specifically, for each PUMA of residence, the fraction of hours supplied to the most popular destination is computed; the table reports statistics on the distribution of such maximum. The first row uses Census 2000, the following three ACS in the reported years, and the last aggregates all years.

Table 3: Distribution of commuting distances

	p50	p75	p90	p99	Max	Mean	N
2000	0	14.68	34.45	132.18	2764.38	14.68	236.6
2005	0	15.66	36.30	150.38	2742.95	15.92	251.1
2006	0	15.40	36.07	151.19	2754.01	15.79	261.0
2007	0	15.67	36.36	160.82	2713.49	16.02	263.4
ALL	0	15.41	36.03	147.80	2764.38	15.62	1,012.0

This table reports selected percentiles in the distribution of commuting distances, in miles. The length of a commuting pattern is computed as the distance between the centroids of the reported PUMAs of residence and of work, using the Haversine formula reported in footnote 22. Centroids of each PUMA have been computed using standard spatial analysis software and requiring that the centroid falls within the PUMA itself. The first row uses Census 2000, the following three ACS in the reported years, and the last aggregates all years. Each observation receives a weight proportional to the yearly hours of labor supplied. The last column reports total hours worked in a year, in billions.

As seen above, the median hour of work commutes within the PUMA of residence. More than 25% of hours are employed in a PUMA different from where the worker lives, which gives confidence that it will be possible to estimate the elasticity of labor supply to differences in wages by comparing relative wages and relative commuting flows. The unrealistically high maximum commuting distances are due to the fact that, for some occupations, the job location is itinerant[23]. In the estimations of the parameters of labor supply, I will repeat all estimates twice, for the complete sample and a truncated one which eliminates the top 1% of commuters by distance: in the restricted sample, results are actually strengthened, in the sense that the implied elasticities of labor supply are higher[24].

As one can expect, commuting flows decrease with distance, and this relation is consistent across Census and ACS sample. Each dot in Figure 9 represents the fraction of hours flowing from any given r to one observed destination j, m_{rj}, in a year, plotted against the distance between the centroids of the two PUMAs. This negative relation fades away after a certain distance (interestingly, around the top 1% cutoff), consistent with the fact that high distances are due to itinerant occupations and do not really reflect daily commuting patterns. Commuting flows decrease with distance with an elasticity of 0.96 in the complete sample, and 1.55 in the truncated sample, after controlling for year fixed effects.

6.2 Employment, hourly income, and geographic distribution of industries

All the analysis will be carried out at the level of hours of work, as the model does not distinguish between worker types, or length of the work week. Hence, the size of a PUMA H_r is computed as the sum across all employed persons living in r of their usual hours of work per week times the number of weeks worked in the preceding year; total employment in a PUMA is the sum across all the workers working in PUMA j of the same quantity[25]. Table 4 reports summary statistics on hours supplied to a given PUMA of work over time.

Table 4: Total hours of work supplied to a PUMA (in million)

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	36.81	69.49	84.33	110.36	165.11	296.30	6,230.56	192.32	1,230
2005	40.19	73.35	86.63	116.46	173.05	313.90	7,062.51	204.13	1,230
2006	41.94	75.43	92.03	119.96	179.74	330.89	7,395.70	212.18	1,230
2007	44.63	73.81	90.63	120.47	179.84	332.39	7,438.68	214.14	1,230
ALL	36.81	72.65	88.14	116.83	174.94	316.28	7,438.68	205.69	4,920

This table reports selected percentiles in the distribution of the hours of work supplied to a PUMA. All values are in millions of hours. The first row uses Census 2000, the following three ACS in the reported years, and the last aggregates all years.

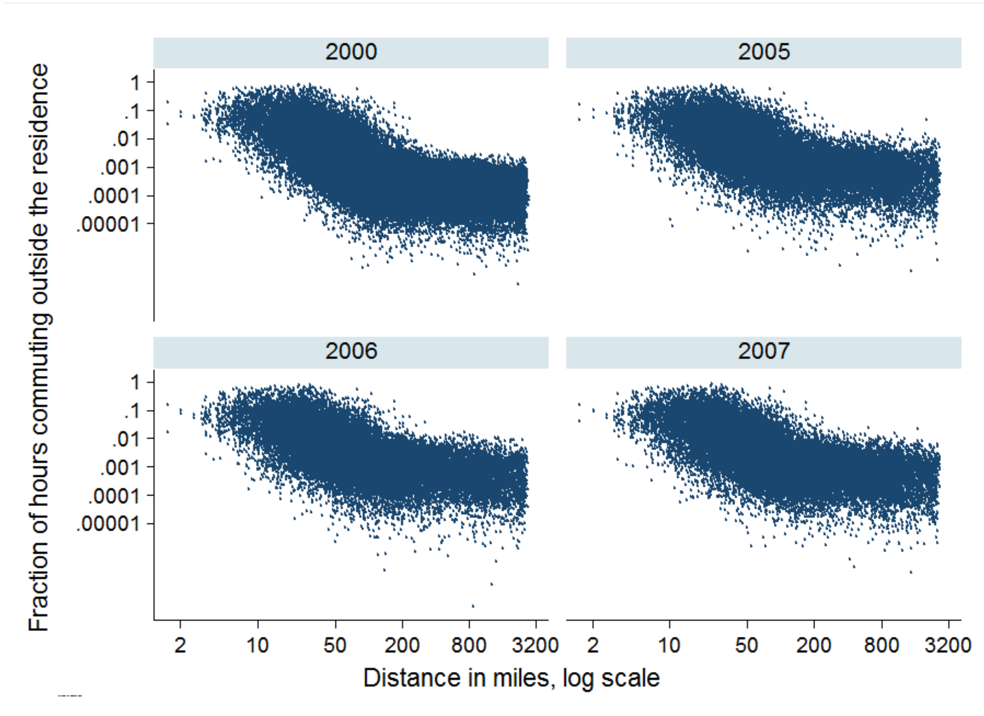
To compute the average income of hours supplied to a location, I sum the total income received by all

[23] The questionnaire asks the job location in the week preceding the interview. I find, for example, that aircraft pilots and engineers, transportation attendances, actors, entertainers and performers, marine engineers and naval architects, are between 10 and 40 times overrepresented in the sample of commuters with distance higher than 1,000 miles, with respect to their proportion in total employment.

[24] The top 1% of commuters report distances above 137 miles. While any selection would be arbitrary, I consider this choice reasonable: the computed distance is between centroids, but the worker may just live on one side of the border between two PUMAs and work on the other; moreover, as Figure 9 will show, the negative relation between bilateral distance between PUMAs and fraction of commuters changes around this distance.

[25] Person weights are used throughout the analysis and we will avoid repeating it in the description.

Figure 9: Commuting and distance



For each year, this figure plots the share of hours commuting from any PUMA of residence r to all PUMAs of work j (excluding r itself) out of the total hours of work supplied by residents in r, againts distance between r and j. The distance between two PUMAs is computed as the distance between their centroids using Haversine's formula, as indicated in footnote 22.

employed people in j (pre-tax wage and salary income, plus pre-tax self-employment income from a business, professional practice, or farm, if present), and divide by the total hours worked there: this value will be used as the empirical counterpart of w_j.[26] All monetary values are in 2007 dollars. Table 5 reports some summary statistics on these hourly income across PUMAs of work.

I close the description of basic patterns in the data summarizing some relevant aspects of the geographic distribution of sectoral activity. To this purpose, I convert the industry of employment of each worker from the Census/ACS codification into 3-digit NAICS in the 2002 classification using the crosswalk made available by Ruggles et al. (2010). The first three digits are identical to the NAICS1997, in which international trade data is also provided. I limit the sector-level analysis to 21 manufacturing sectors (NAICS 311-339) and pool all the others sectors into a residual service, non-tradeable sector. In line with other studies (e.g. Pierce and Schott, 2012, Autor, Dorn and Hanson, 2013), this data shows a shrinking share of employment in manufacturing, from about 15% of total hours worked in 2000 to about 12% in 2007. Table 6 shows (Panel A) that half the PUMAs have employment in at least 15 of the 21 manufacturing sectors (19 of these sectors, when considering the

[26] We have considered using only salaried employees. This would come at a price of excluding part of employment from our analysis, while the model does not distinguishes between different forms of income. Since the correlation between hourly income and hourly wage at PUMA of work level is 0.9951, it seems reasonable to proceed with a more complete picture of total hours of work supplied.

Table 5: Average hourly income by PUMA of work (dollars)

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	13.59	15.94	17.05	19.10	21.95	25.01	38.27	19.89	1,230
2005	12.45	15.68	16.93	19.10	22.11	25.68	48.05	20.02	1,230
2006	13.14	15.51	16.79	18.95	21.71	25.12	38.88	19.76	1,230
2007	13.39	15.83	17.19	19.36	22.05	25.59	44.53	20.12	1,230
ALL	12.45	15.72	16.99	19.11	21.95	25.38	48.05	19.95	4,920

This table reports selected percentiles in the distribution of the hourly income of hours of work supplied to a PUMA. Hourly income in a PUMA is computed as the total income received by all employed people in a PUMA (pre-tax wage and salary income, plus pre-tax self-employment income from a business, professional practice, or farm, if present), divided by the total hours supplied to the same PUMA. All values are 2007 dollars. The first row uses Census 2000, the following three ACS in the reported years, and the last aggregates all years.

CENSUS data). The other half typically has employment in only some of them. Panel B of the same table also documents that PUMAs vary considerably in the reliance of their employment on the manufacturing sector: moving from the 10^{th} to the 90^{th} percentile increases the prevalence of manufacturing employment by a factor of 5.

Table 6: The manufacturing sector across PUMAs

Panel A: Number of active manufacturing sectors across PUMAs of work

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	12	17	18	19	20	21	21	19	1,230
2005	2	11	13	15	17	19	21	15	1,230
2006	5	10	13	15	17	19	21	15	1,230
2007	3	11	13	15	17	19	21	15	1,230
ALL	2	11	14	16	19	20	21	16	4,920

Panel B: Share of hours employed in manufacturing

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	1.76	6.77	10.39	16.19	22.86	28.97	49.56	17.15	1,230
2005	0.61	5.53	8.81	14.08	19.92	25.63	50.25	15.05	1,230
2006	1.32	5.39	8.70	13.56	19.75	25.17	48.24	14.77	1,230
2007	1.08	5.32	8.30	13.28	19.10	24.39	49.12	14.30	1,230
ALL	0.61	5.68	8.95	14.17	20.55	25.97	50.25	15.32	4,920

This table reports selected percentiles in the distribution of the manufacturing sector (NAICS 311-339) across PUMAs of work. Panel A reports the number of different 3-digit NAICS sectors active in manufacturing across PUMAs. Panel B reports the percentage of hours employed in manufacturing over the total employment in a PUMA. In each Panel, the first row uses Census 2000, the following three ACS in the reported years, and the last aggregates all years.

Table 7 shows evidence on sector concentration: Panel A shows that while for the median manufacturing sector employment is reported in (typically) 972-982 locations (1,204 when considering CENSUS data), a quarter of the sectors is only present in half of the PUMAs, and 5% of the sectors is only present in a third of locations. Even if sectors are absent from a location, we will see in a counterfactual example below that those locations are nonetheless impacted by (even small) episodes of trade integration. Employment in a sector is however distributed very unevenly even across location which report some activity in it: Panel B of the same table shows in fact that for the median sector, the 61 most important contributing locations in terms of employment (i.e., 5% of the PUMAs) account for 40% of employment in the sector. The top 5% of PUMAs never represent less than about 30% of employment of any sector, and can reach two-thirds of employment in some extreme cases.

Table 7: Concentration of manufacturing activity

Panel A: Number of locations where a sector is active

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	655	937	1,104	1,204	1,222	1,227	1,228	1,124	21
2005	241	454	601	972	1,115	1,146	1,171	864	21
2006	244	454	582	980	1,094	1,145	1,188	868	21
2007	244	455	583	982	1,115	1,152	1,175	873	21

Panel B: Share of employment of a sector in the top 5% of PUMAs

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	27.40	29.23	31.85	38.93	54.38	56.14	64.96	41.81	21
2005	28.99	30.48	32.66	39.39	57.07	65.52	66.93	44.20	21
2006	29.37	30.67	30.92	39.77	57.44	64.75	68.35	43.79	21
2007	30.23	30.53	32.25	40.83	56.02	64.14	68.23	43.79	21

This table reports selected percentiles in the distribution of the concentration of different manufacturing sectors (NAICS 311-339). Panel A refers the number of different PUMAs where a given sector has positive employment. Panel B refers to the percentage of employment of a sector accounted for by the largest 5% of PUMAs in terms of employment in the sector (61 locations). In each Panel, the first row uses Census 2000, and the following three ACS in the reported years.

These facts combined imply that changes in trade frictions affecting only some sectors affect different PUMAs asymmetrically, and with a variable degree of intensity.

Having described the data sources and their main features, I now show how they can be used to identify the parameters of labor supply and labor demand across PUMAs.

7 Identification and estimation

The main parameters of the model can be identified by variation in separate dimensions of data on the US economy.

7.1 Elasticity of labor supply across locations

One key piece of information to properly perform counterfactuals in this economy is the estimation of the set of parameters μ_r, which regulate the elasticity of commuting flows (and hence of labor supply) to changes in relative wages. From (2), one can compute

$$\log \frac{m_r(j)}{m_r(r)} = \frac{1}{\mu_r} \log \frac{w_j}{w_r} - \frac{1}{\mu_r} \log \frac{d_{rj}}{d_{rr}}$$

for any pair of origin-destinations observed in the data. The intuition is the following: when μ_r is very small, even small differences in relative wages are reflected in large differences in relative commuting flows; when μ_r is very large, on the other hand, idiosyncratic preferences are the main factor determining job destination, and even large differences in relative wages do not impact relative commuting flows significantly.

The fact that different PUMAs have different geographical size and different levels of infrastructure development requires an estimation of μ specific for each residence location: hence, I will model μ_r to be a (reduced form) function of the the total length of major highways, minor highways, major roads and railroads present in each of the PUMA, and the total area of each PUMA, which I have computed directly using digital maps from ESRI (2010); hence,

$$\mu_r = \eta_0 \times Roads_r^{\eta_1} \times Area_r^{\eta_2}$$

I decompose the commuting cost faced by residents in r towards j as a function of the product of total roads' length in r and j, the product of the total areas of r and j, and the average opportunity cost of commuting time of workers from r to j. To compute this last piece of information, I multiply the hourly income of each commuter from r to j times twice the commuting time that this worker reports in the data, and compute a weighted average of this cost across all commuters. Hence,

$$d_{rj} = OppCost_{rj}^{\gamma_1} \times (Roads_r \ Roads_j)^{\gamma_2} \times (Area_r \ Area_j)^{\gamma_3} \times \tilde{\varepsilon}_{rj}$$

where $\tilde{\varepsilon}_{rj}$ is a disturbance term. Data on the fraction of hours of work supplied commuting from r to any other j and on the average hourly income paid in location j is computed from ACS as indicated above.

After substituting these expressions,

$$\begin{aligned}\log \frac{m_r(j)}{m_r(r)} &= \alpha + \eta_0^{-1} Roads_r^{-\eta_1} Area_r^{-\eta_2} \log \frac{w_j}{w_r} - \eta_0^{-1} Roads_r^{-\eta_1} Area_r^{-\eta_2} \gamma_1 \log \frac{OppCost_{rj}}{OppCost_{rr}} \\ &\quad - \eta_0^{-1} Roads_r^{-\eta_1} Area_r^{-\eta_2} \gamma_2 \log \frac{Roads_j}{Roads_r} - \eta_0^{-1} Roads_r^{-\eta_1} Area_r^{-\eta_2} \gamma_3 \log \frac{Area_j}{Area_r} + \varepsilon_{rj} \qquad (21)\end{aligned}$$

where

$$\begin{aligned}\alpha &= -\eta_0^{-1} Roads_r^{-\eta_1} Area_r^{-\eta_2} E \log \frac{\tilde{\varepsilon}_{rj}}{\tilde{\varepsilon}_{rr}} \\ \varepsilon_{rj} &= -\eta_0^{-1} Roads_r^{-\eta_1} Area_r^{-\eta_2} \left[\log \frac{\tilde{\varepsilon}_{rj}}{\tilde{\varepsilon}_{rr}} - E \log \frac{\tilde{\varepsilon}_{rj}}{\tilde{\varepsilon}_{rr}}\right]\end{aligned}$$

Of course, according to the model equilibrium wages do depend on commuting costs d_{rj} and d_{rr} (and hence on $\tilde{\varepsilon}_{rj}/\tilde{\varepsilon}_{rr}$) so that a simple regression would suffer from the correlation of $\log \frac{w_j}{w_r}$ with the unobservable. The model however suggests that (functions of) the measure of agents living in a location r, H_r, or living around location r but not in r should be valid instruments, as they affect the relative wage through equilibrium relations but should be uncorrelated with ε_{rj} (which is only function of parameters and i.i.d errors). I will use as instruments the log of four ratios: total labor force (not just employment) living in j divided by r; total labor force living in PUMAs whose centroid is less than 200 miles away from j (but not in j) divided by the same quantity for r; and the same two ratios, computed using total population 16 years old and older rather than total labor force.

I estimate this model with GMM, clustering errors at the level of PUMA of residence (which leaves us 1,230 clusters) only for data in the year 2000 (i.e., with Census data), for the years 2005-2007 (i.e., with ACS data), and for the whole sample; I repeat these estimations using all recorded commuting patterns first, and then truncating away recorded origin-destinations whose PUMA centroids are more than 137 miles apart (i.e., the top 1% of commuters by distance).[27] The results are presented in Table 8.

Table 8: GMM estimation of parameters of local labor supply

	All distances			Truncated distances		
	Only 2000	Only 2005-07	All years	Only 2000	Only 2005-07	All years
η_0	3.48***	4.47***	4.47***	5.34	4.84	4.58
	(0.90)	(1.30)	(1.04)	(4.16)	(3.70)	(3.43)
η_1	-0.44***	-0.41***	-0.43***	-0.77***	-0.71**	-0.72***
	(0.08)	(0.07)	(0.07)	(0.25)	(0.29)	(0.27)
η_2	0.23***	0.21***	0.22***	0.42***	0.40***	0.40***
	(0.05)	(0.04)	(0.04)	(0.12)	(0.15)	(0.14)
γ_1	-0.99***	-2.01*	-1.49**	-0.17	-0.37	-0.29
	(0.32)	(1.05)	(0.60)	(0.21)	(0.38)	(0.29)
γ_2	0.17***	0.27**	0.23***	0.03	0.04	0.04
	(0.05)	(0.13)	(0.09)	(0.03)	(0.04)	(0.03)
γ_3	-0.12***	-0.20*	-0.16**	-0.02	-0.03	-0.03
	(0.04)	(0.10)	(0.07)	(0.02)	(0.03)	(0.03)
α_0	4.62***	6.29**	5.62***	1.70**	1.84*	1.78**
	(1.01)	(2.75)	(1.77)	(0.67)	(1.07)	(0.87)
N	64,908	97,018	161,926	32,543	65,237	97,780

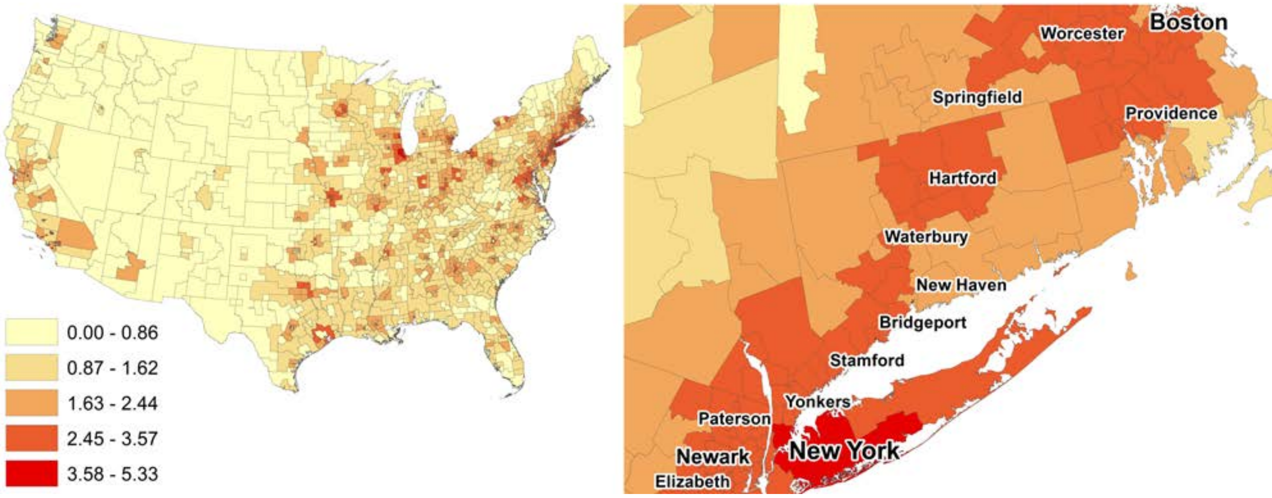
This table shows GMM estimates for eq. 21. Each column reports the results of a particular GMM estimate: columns differ only according to the sample used. The first three columns use all observed bilateral commuting flows; the last three only bilateral commuting flows between PUMAs whose centroids are less than 137 miles away from each other (i.e., exclude the top 1% of commuters). Within each set of three columns, the first uses only commuting data from then Census 2000, the second only data from ACS 2005-2007, and the third all data together. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$. Standard errors in parentheses.

The main coefficients of interests η_1 and η_2 are very precisely estimated and are stable across subsamples. As expected, a denser transportation network facilitates the response of commuting flows to differences in wages ($\eta_1 < 0$, so that more roads increase $1/\mu_r$ and make commuting flows more elastic), while a larger PUMA area makes the measured elasticity smaller ($\eta_2 > 0$, so that in a larger PUMA $1/\mu_r$ is smaller and commuting flows are less responsive to differences in wages). Moving from the complete to the truncated sample takes away the significance of the other coefficients, but actually increases the sensitivity of μ_r to roads and PUMA areas[28]. Table 9 below shows summary statistics for the distributions of 1,230 μ_r parameters estimated using all years

[27] I also repeat the same estimation introducing time dummies for the constant, which allows for different average $\tilde{\varepsilon}_{rj}/\tilde{\varepsilon}_{rr}$ over time, with essentially unchanged results. These estimates are reported in the Appendix, Table 15.

[28] Both coefficients increase in absolute values. A J test for overidentifying restrictions is not passed at 5% significance (but passed at 1%, as p-values are roughly between 1.5 and 3.8%) in the unrestricted sample, but always passed (p-values are always above 0.35) in the restricted sample. For this reason, we will use the estimates of μ_r from the restricted sample in our counterfactual exercises.

Figure 10: Elasticity of labor supply to a PUMA to its own wage



This Figure shows the elasticity $\varepsilon(L(j), w_j)$ (eq. (5)) over the US territory (left panel) and zooming on a portion of the East Coast (right panel). Elasticities are computed using observed data on commuting flows in 2005 and estimates of μ_r from the restricted sample reported in table 9.

for the unrestricted and restricted samples[29].

Table 9: Distribution of estimates of μ_r

	Min	p10	p25	p50	p75	p90	Max	Mean	N
Unrestricted	0.44	0.70	0.77	0.82	0.88	0.95	1.36	0.82	1,230
Restricted	0.13	0.27	0.31	0.35	0.40	0.46	0.88	0.36	1,230

This table reports selected percentiles in the distribution of the estimates of the parameter μ_r implied by the estimation of eq. 21 reported in table 8. The first line refers to estimates based using the whole sample of commuting patterns (third column), and the second line to estimates based on the restricted sample (sixth column).

These estimates of μ_r can be plugged in eq. (5) to estimate the elasticities of labor supply to location j with respect to own wages and wages in any other locations. We report below some pictures representing these estimates.[30]

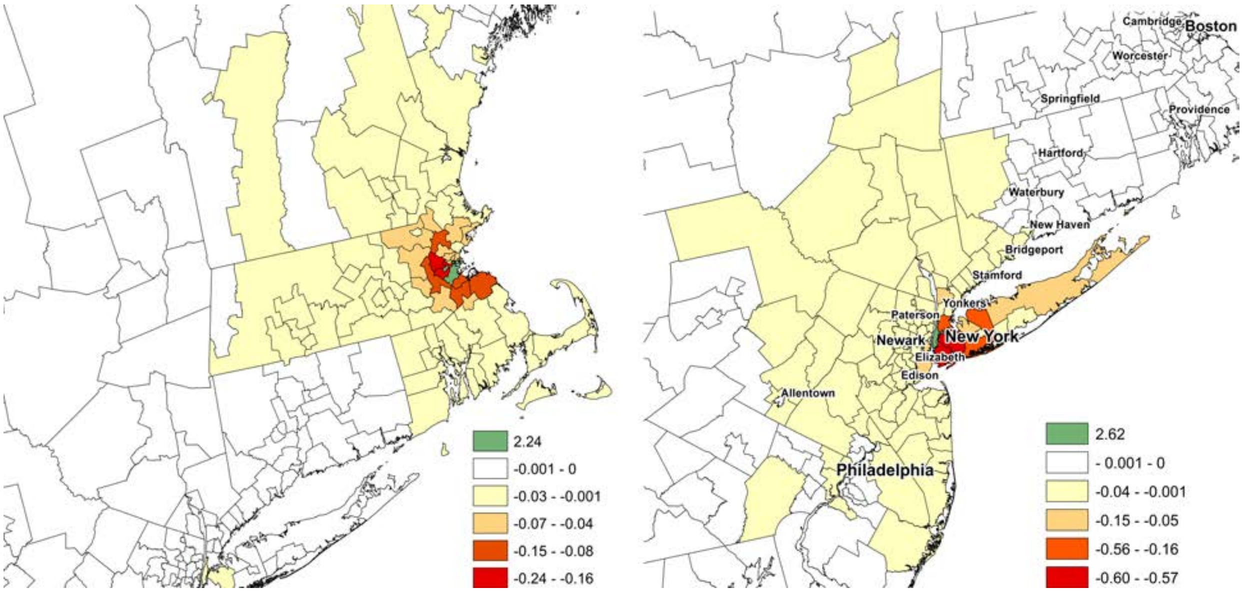
Figure 10 plots the elasticity of labor supply to a location j with respect to the prevailing wage in the same area, $\varepsilon\left(L\left(j\right), w_j\right)$. The left panel plots these elasticities for all 1,230 PUMAs, while the right one zooms in on the New York - Boston area. As can be seen, the estimated elasticities are quite heterogeneous.

Figure 11 plots the elasticity of labor supply to PUMA 2503300 (Boston, MA) and PUMA 3603800 (Manhattan) to wages in the same area (positive, in green) and to neighboring areas, truncating those elasticities smaller than 0.1% in absolute value. Interestingly, wages in Manhattan may affect what happens in Boston (by

[29] All the estimated μ_r in the unrestricted sample are significantly different from zero at less than 1% significance; about 95% of the μ_r in the restricted sample are significantly different from zero at 5% significance. PUMAs with μ_r not significantly different from zero are on average 3.5 times smaller than the average PUMA, and have a 2.6 times denser road network (measured in miles of roads per square mile of PUMA area).

[30] In these computations, we use the estimates from the restricted sample.

Figure 11: Elasticity of labor supply to a PUMA to wages around it



This figure shows the elasticities$\varepsilon(L(j), w_{j'})$ for PUMAs 2503300 (Boston, MA) and 3603800 (Manhattan, NY). Elasticities are computed using observed data on commuting flows in 2005 and estimates of μ_r from the restricted sample reported in table 9.

diverting away workers who can commute to both destinations). It is clear that one can draw a similar picture for all the PUMAs between these two urban areas: local labor markets continuously overlap over the territory.

Once we have obtained the estimates from eq. 21, we can recover all the d_{rj} (with the normalization $d_{rr} = 1$). To run counterfactual simulations, we still need to find parameters values for the demand side of the model, which is the subject of the following subsection.

7.2 Parameters of labor demand

The crucial parameters to estimate in the labor demand equation (14) are the dispersion parameter θ and the technology parameters $T_j^{(s)}$, both determining the shares of expenditure of the home economy $\pi_j^{(s)}$ and the foreign economy $\pi_{*j}^{(s)}$ falling on goods of sector s produced in location j.

Since identification will come from time variation, I introduce time indices in the subsequent expressions. To reduce the dimensionality of the problem, I will assume that $T_{jt}^{(s)} = T_j T^{(s)} e^{\varepsilon_{jt}^{(s)}}$, i.e., the productivity of a location in a sector is coming from characteristics specific to a location but common across sectors, T_j, and characteristics specific to a sector but common across locations, $T^{(s)}$, plus an idiosyncratic component log-normally distributed, $\varepsilon_{jt}^{(s)}$. Consider the total income of workers active in sector s and working in location j,

$W_{jt}^{(s)}$. From eq. (14), and using the expressions for $\pi_{jt}^{(s)}$ and $\pi_{*jt}^{(s)}$,

$$\begin{aligned} W_{jt}^{(s)} &= \pi_{jt}^{(s)}\alpha^{(s)}X_t + \pi_{*j}^{(s)}\alpha^{(s)}X_{*t} \\ &= w_{jt}^{-\theta}\alpha^{(s)}\left[\frac{X_t}{\Phi_t^{(s)}} + \tau^{(s)-\theta}\frac{X_{*t}}{\Phi_{*t}^{(s)}}\right]T_jT^{(s)}e^{\varepsilon_{jt}^{(s)}} \implies \\ \log W_{jt}^{(s)} &= \log\alpha^{(s)}T^{(s)} + \log T_j - \theta\log w_{jt} + \log\left[\frac{X_t}{\Phi_t^{(s)}} + \tau^{(s)-\theta}\frac{X_{*t}}{\Phi_{*t}^{(s)}}\right] + \varepsilon_{jt}^{(s)} \qquad (22) \end{aligned}$$

which says that in principle, θ can be estimated with a regression of the log wage bill paid to a sector in a location on sector, location, and sector-year fixed-effects and the hourly compensation received by workers in a given location. The intuition is the following: comparing all locations that have the same sector s active, if the strength of comparative advantage is low (i.e., θ is high and goods are homogeneous in terms of productivity) a slightly higher prevailing wage in j would make sector s less competitive in a wide range of goods everywhere, and reduce heavily total labor demand (and hence total wage bill) for workers in this sector. On the contrary, if productivities tend to be dispersed, it takes a relatively larger increase in w_j to reduce the range of goods for which j is the lowest cost producer by the same amount. Note that a similar regression could be run with the total number of hours worked in sector s location j, $H_j^{(s)}$, on the left-hand side, and the coefficient on wages would then be $-(\theta+1)$ rather than $-\theta$: while the model forces the wage to be identical across sectors in a location, this will not be true in general in the data, and hence the two approaches are not necessarily equivalent. We will explore both alternatives.

One concern with this regression is of course endogeneity: the idiosyncratic shocks $\varepsilon_{jt}^{(s)}$ is part of the productivity $T_{jt}^{(s)}$, is not observed, and wages w_{jt} do depend in equilibrium on it. It could be that a higher $\varepsilon_{jt}^{(s)}$ increase both the wage in a location and the total labor demand in sector s, thus biasing the estimate of the coefficient $-\theta$ upwards, towards positive values. To address this issue, we again note that the resident population and labor force in location j or around it are valid instruments according to the model. We will exploit variation in the labor force and resident population over time within location-sector cells to identify θ. we run a 2-stages least square of the form

$$\begin{aligned} \log W_{jt}^{(s)} &= \log\alpha^{(s)}T^{(s)} + \log T_j + \delta_t^{(s)} - \theta\log w_{j,t} + \varepsilon_{j,t}^{(s)} \qquad (23) \\ \log H_j^{(s)} &= \log\alpha^{(s)}T^{(s)} + \log T_j + \delta_t^{(s)} - (\theta+1)\log w_{j,t} + \varepsilon_{j,t}^{(s)} \qquad (24) \end{aligned}$$

where $\log\alpha^{(s)}T^{(s)}$ is a sector dummy, $\log T_j$ is a location dummy, $\delta_t^{(s)}$ is a year-sector dummy, and instrumenting $w_{j,t}$ with (combinations of) the log of labor force and resident population in j and around it. The results of these estimations are reported in Table 10.

The two columns report results when the dependent variables are total wage bill and total hours worked, respectively. The first line reports the value of $-\theta$ with simple OLS estimates of these equations. As can be seen, the coefficient is positive (implying a negative value for θ): unobserved productivity in a sector/location generates a positive correlation between hourly wage and total wage bill. Once we use the log of total population

Table 10: Coefficient -θ in IV estimation of eq. 23 and eq. 24

	Dep. var: wage bill		Dep. var: tot. hours	
	$-\theta$	N	$-\theta-1$	N
OLS	0.62***	78,284	0.91	78,325
	(0.12)		(0.10)	
IV: only local	-7.80***	78,284	-8.80***	78,325
	(1.79)		(1.61)	
IV: local and 50 miles	-4.92***	70,266	-5.42***	78,298
	(1.37)		(1.22)	
IV: local and 200 miles	-4.05***	70,284	-4.43***	78,325
	(1.49)		(1.32)	

This table shows estimates of equation 23 (left column, reporting $-\theta$) and equation 24 (right column, reporting $-\theta-1$). The first row shows results for an OLS regression of the dependent variable (total wage bill or total hours in location j and sector s) on sector, locations, and sector-year fixed effects. The second row show IV estimates of the same equations, where the instrument is the total population 16 y/o and older and labor force living in j. The third and fourth rows add population 16 y/o and older and labor force in PUMAs whose centroid is within 50 miles, or within 200 miles, from the centroid of j, respectively. * $p<0.1$; ** $p<0.05$; *** $p<0.01$. Standard errors in parentheses.

and labor force (second row) as an instrument, however, the coefficient is negative and precisely estimated. The second and third rows add log population and labor force within 50 miles, and within 200 miles of the PUMA, respectively. As we would expect, adding instruments which are arguably less able to shift labor supply makes the endogeneity problem somewhat more apparent, and decreases the estimates of θ. Note also that the coefficients on hours are typically below (but not exactly 1 less than) the coefficient on wages, confirming the importance of within-location, between-sectors variability in wages[31]. In what follows, we will use the benchmark value of $\theta = 7.80$.

With these estimates, we are in a position to estimate the technology parameters. The values T_j, $\alpha^{(s)}T^{(s)}$, are estimated directly, and $\varepsilon_j^{(s)}$is a residual for each sector-location-year. To calibrate shares $\alpha^{(s)}$ in the utility function, we start by dividing expenditure in manufacturing and non-manufacturing using Dekle Eaton and Kortum (2008) value of 0.78 for the US share of expenditure on non-tradeable. Hence, $\alpha^{\bar{M}} = 0.78$, while the expenditure share on manufacturing is set to 0.22. We further merge the NBER productivity database and NBER International Trade US imports and exports from 2000 to 2006 and aggregate the values at the 3-digit NAICS to obtain the share of expenditure by sector as

$$\alpha^{(s)} = 0.22 \times \frac{\textit{Value of production}\,(s) + \textit{Imports}\,(s) - \textit{Exports}(s)}{\sum_{s' \in S_M} \textit{Value of production}\,(s') + \textit{Imports}\,(s') - \textit{Exports}(s')}$$

Hence, we can obtain values for $T^{(s)}$. Note now that we can read in the data the import penetration in sector

[31] The number of observations when instrumenting with population and labor force within 50 miles drops as some PUMAs have no other centroids within 50 miles of theirs, resulting in a missing value ($\log(0)$) for the instrument. The number of observations comparing the two dependent variables changes as for a small number of PUMA-years the average hourly income is negative, resulting in a missing value (log of a negative number) for the dependent variable.

$s:$

$$\pi_*^{(s)} = \frac{\text{Imports}\,(s)}{\textit{Value of production}\,(s) + \textit{Imports}\,(s) - \textit{Exports}\,(s)}$$

The model however says, using $T_j^{(s)} = T_j T^{(s)} e^{\varepsilon_j^{(s)}}$,

$$\pi_*^{(s)} = \frac{T_*^{(s)} \left(\tau^{(s)} w_*\right)^{-\theta}}{T_*^{(s)} \left(\tau^{(s)} w_*\right)^{-\theta} + \sum_{j \in \mathcal{R}} T_j^{(s)} w_j^{-\theta}} = \frac{T_*^{(s)} \left(\tau^{(s)} w_*\right)^{-\theta}}{T_*^{(s)} \left(\tau^{(s)} w_*\right)^{-\theta} + T^{(s)} \sum_{j \in \mathcal{R}} T_j e^{\varepsilon_j^{(s)}} w_j^{-\theta}}$$

and solving for $T_*^{(s)}$,

$$T_*^{(s)} = \frac{\pi_*^{(s)}}{1 - \pi_*^{(s)}} \frac{\sum_{j \in \mathcal{R}} T_j e^{\varepsilon_j^{(s)}} w_j^{-\theta}}{\left(\tau^{(s)} w_*\right)^{-\theta}} T^{(s)}$$

Up to the wage in the foreign country, everything on the right-hand side is known. To find a value for the average hourly income in the rest of the world, we resort to the Penn World Tables, version 8. We want to compute a wage for the foreign economy that is consistent with the data on employment in US in the Census/ACS. In the years 2000-2007, US represents about 5% of the world employment (and also of the world population): hence, $H = 0.05\,(H + H_*)$, where H is the total hours worked on average by the US employment. I then set $H_* = H \times 0.95/0.05 = 4,807$ billion hours. Similarly, US represents about 22% of world GDP, i.e., $X = 0.22\,(X + X_*)$; hence, $w_* = X_*/H_* = (X/0.22 - X)\,/\,(H/0.05 - H) = (\bar{w}H/0.22 - \bar{w}H)\,/\,(H/0.05 - H) = 4.13$, where $\bar{w}$ is the average hourly income of US. With w_* in hand, we can back out $T_*^{(s)}$.

Finally, we use the cost of freight, insurance, and tariffs paid at the border for US imports from the same International Trade dataset to impute values for the trade costs $\tau^{(s)}$ at each of these sectors.

8 Counterfactual simulations

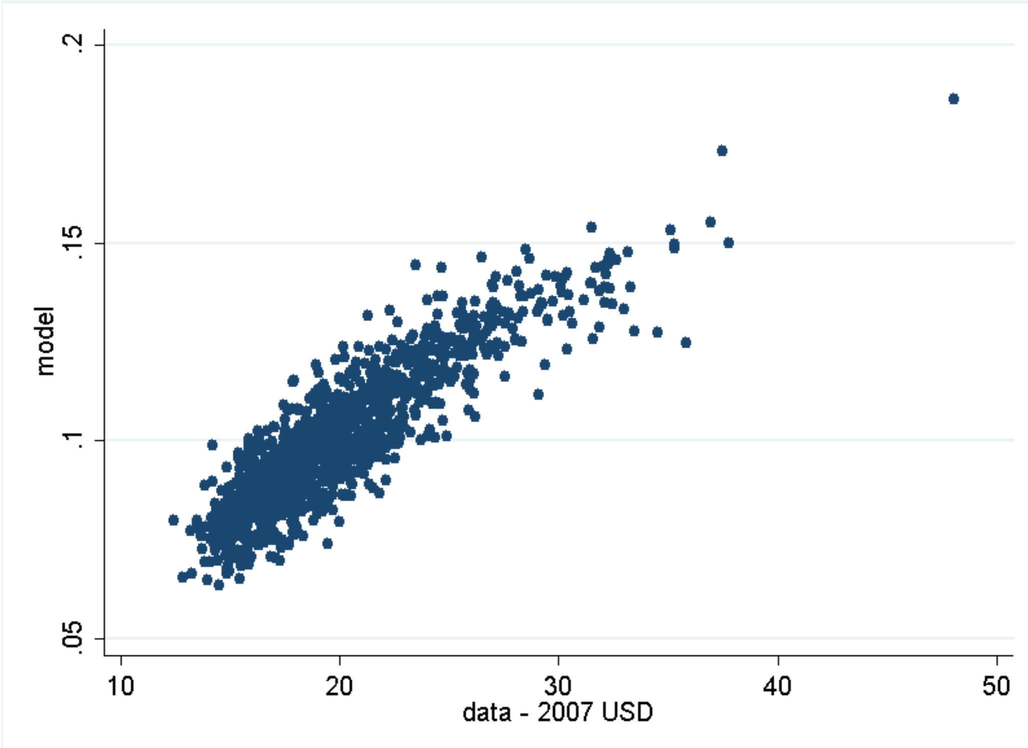
With the parameters of the model in hand, I simulate the equilibrium value of wages in 2005. Figure 12 shows that across PUMAs, simulated wages and wages in the data exhibit a very high correlation.[32] In the following subsections, I first explore the response of the US economy to the elimination of trade frictions in a single sector, with the purpose of emphasizing linkages between areas impacted directly and indirectly by the shock and of quantifying the equilibrium value of different components of the transmission. I then turn to study the consequences of an increase in the manufacturing productivity of the rest of the world, focusing on employment and wages outcomes, and the relation between changes in local nominal vs. real wages.

8.1 Liberalization in a single sector

In this illustrative simulation, I study the impact of a complete elimination of bilateral trade frictions in NAICS sector 334, "Computer and Electronics Product Manufacturing" (CEP) across PUMAs. I will refer to this change as a "trade liberalization" for brevity. This NAICS sector is, among 3-digit NAICS, the one with lowest bilateral trade frictions (tariff plus freight and insurance), $\tau = 1.0206$ on average between 2000 and 2006, as

[32] This fact is reassuring in light of the possible multiplicity of equilibria, as it signals that the equilibrium to which the simulation converges is similar to the equilibrium being observed in the data.

Figure 12: Simulated and measured data across PUMAs



This figure shows a scatterplot of the hourly income across PUMAs in 2005 and the equilibrium wages in the simulated model using parameters estimated from the data. Wages in the data are in 2007 dollars.

computed with the CIF/FOB ratio from the NBER-International Trade Database. It is also one where the revealed comparative advantage of US vs. the rest of the world is among the highest (Costinot, Donaldson, Komunjer 2012).[33] This sector accounts for 9.1% of manufacturing employment and 1.2% of total employment, in terms of hours, and it is relatively highly concentrated: the top 5% of PUMAs account for 57% of total employment in the sector, placing the sector around the 75^{th} percentile across sectors (see again Table 7); the remaining 95% of PUMAs do not contribute more than 0.3% of hours each; 250 PUMAs report exactly zero employment in this sector.

To study the impact of a liberalization I simulate the model twice: first with the parameters estimated and calibrated as described above, and then setting $\tau^{(334)} = 1$ rather than 1.0206. Each simulation computes the equilibrium wages paid to workers employed in each location and hence all other outcomes. I then compare outcomes before and after the elimination of the trade frictions.

Table 11 shows that in the median PUMA, this liberalization implies an increase of the average wage paid to workers of 0.221%, and a movement from the minimum to the maximum impact covers about 40% of this median. Once we condition on whether the PUMAs have positive employment in the sector or not, we find that the order of magnitude of the impact is comparable between the two categories. In other words, while the distribution of impact on PUMAs not active in CEP is slightly shifted to the left, being inactive in the sector is

[33] NAICS 334 corresponds roughly to sectors 30-33, "Electrical and Optical Equipment" in the classification ISIC rev 3.1 in their paper.

not enough to be shielded from the consequences of this small liberalization, and the impact is typically almost as big as the one measured in locations active in CEP. These considerations are important for empirical studies who build measures of exposure to trade based on local employment characteristics: in this case, exposure to trade is zero by construction where CEP is not present, and yet the impact of trade is of the same order of magnitude as where CEP is active.

Table 11: Percentage change in prevailing wage across PUMAs

	Min	p10	p25	p50	p75	p90	Max	Mean	N
All PUMAs	0.188	0.201	0.218	0.221	0.225	0.229	0.274	0.220	1,230
Positive empl.	0.213	0.218	0.220	0.223	0.226	0.230	0.274	0.223	980
Zero empl.	0.188	0.192	0.196	0.202	0.209	0.215	0.223	0.203	250

This table shows the distribution of the impact of an elimination of trade frictions in NAICS 334, "Computer and Electronics Product Manufacturing", across PUMAs. The first row reports the distribution of impacts across all PUMAs, the second across PUMAs active in sector 334, the third across PUMAs inactive in sector 334.

Where do these effects come from, and in particular, why are locations not active in CEP impacted nonetheless? We can decompose the change in wage following Proposition 5 exactly in its four components. While Proposition 5 applies to infinitesimal changes, we can always write:

$$dw_j \equiv w_j\left(\tau^{334}=1\right) - w_j\left(\tau^{334}=1.0206\right) = -\int_1^{1.0206} \frac{\partial w_j\left(\tau^{334}\right)}{\partial \tau^{(334)}} d\tau = -\int_1^{1.0206} \frac{w_j\left(\tau^{334}\right)}{\tau^{(334)}} \varepsilon\left(w_j\left(\tau^{334}\right), \tau^{(334)}\right) d\tau$$

and then use the Proposition to decompose $\varepsilon\left(w_j\left(\tau^{334}\right), \tau^{(334)}\right)$ in separate terms. I simulate the model for a fine grid of values over τ, and approximate this integral with a sum of finite terms[34]; by construction, such a sum is equal to $w_j\left(\tau^{334}=1\right) - w_j\left(\tau^{334}=1.0206\right)$.

Table 12 shows the average importance of each term, for all PUMAs, and distinguishing them by whether or not the PUMA is active in the sector (Figure 21 in the Appendix show the distribution of each of these components across PUMAs). For the average PUMA j with no employment in CEP, a little more than 60% of the effect occurs because connected locations (whether they are active in CEP or not) experience an equilibrium change in wage, and this change has an impact on local labor supply and local labor demand; one-eight of the total effect happens because locations which are far enough from j to affect its local labor supply, affect nonetheless its local labor demand via overlaps in the set of active sectors; the remaining one-fourth is attributable to international trade rebalancing. Among locations with positive employment, only 8% of the total effect is attributable to the direct effect of removal of bilateral frictions on labor demand in a location. If one was to relate these observed changes to changes in trade exposure,

As a final note, it is interesting to compare these predictions with those of a model where each labor market is isolated. To perform this exercise, I manipulate the matrix of commuting possibilities so that residents in any r can only work in r itself, and repeat the analysis above: I compute the change in wages dw_j^{NC} following a reduction in $\tau^{(334)}$ from 1.02 to 1. The quantity $\ln\left(dw_j^{NC}/dw_j\right)$ measures the percentage overprediction (or

[34] In particular, the interval $[1, 1.0206]$ is split in 10 equally long segments.

Table 12: Decomposition of the effects of a liberalization in CEP

	Zero Empl. in CEP	Positive Empl. in CEP	Overall
Direct	0	8.0	6.4
Connected LM	61.6	60.5	60.7
Unconnected LM	12.3	8.5	9.3
Trade Balance	26.1	23.0	23.6
Total	100	100	100

This table shows the contribution of each of the terms in Proposition 5 to the change in wages across PUMAs following a reduction of trade barriers in CEP from 1.0206 to 1. The first column reports the average for PUMAs with zero employment in CEP, the sectond column for those with positive employment in CEP, and the third for all PUMAs.

underprediction, if negative) in wage changes that one would incur into when considering each labor market in isolation, while the true data-generating process includes commuting possibilities. Figure 13 shows that a model with no commuting would over-predict wage changes in locations active in CEP, while under-predicting changes in regions inactive in it. This is indeed intuitive. In the data generated by the true model, large increases in labor demand in a location can be partially accommodated by labor flowing in from neighboring places. If we shut down this possibility, the equilibrium increase in wage in locations expanding relatively more (less) must be stronger (weaker) to bring the equilibrium back in each local labor market.[35]

8.2 Foreign productivity growth

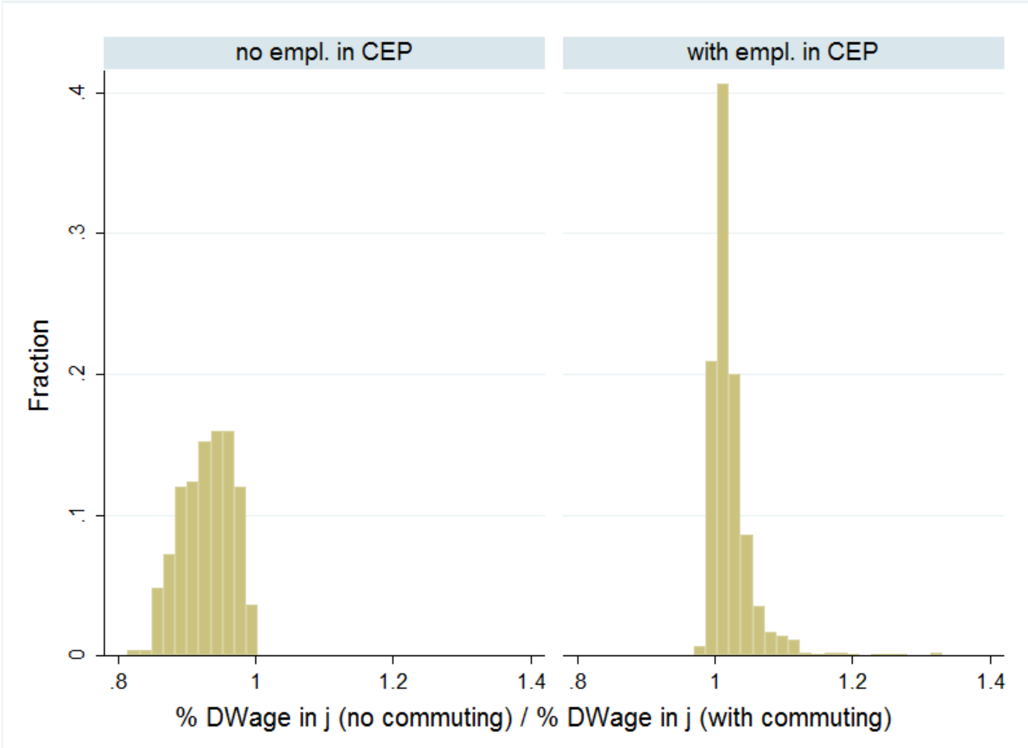
The level of trade barriers and of transportation costs that can be inferred from the CIF/FOB ratio in the NBER Trade database is typically very small. On the other hand, a central feature of the world economy in recent decades is the spectacular growth in the manufacturing productivity of developing countries. Productivity growth in China's manufacturing sector, for example, has been twice the growth in the US manufacturing sector for almost two decades. Hsieh and Ossa (2011) and Di Giovanni, Levchenko and Zhang (2013) study the spillover of such growth across the rest of the world' welfare, and Autor, Dorn and Hanson (2013) study the impact of this surge across local labor markets in US. In this subsection, I start from the US in 2005 and study the consequences of a tripling in the productivity of the rest of the world's manufacturing sector.

Table 13 shows summary statistics on the distribution of PUMA-level changes in selected variables. As productivity in the rest of the world increases, two things happen: first, the share of the rest of the world's products in any market's expenditure increases, thus decreasing labor demand in the manufacturing sector across PUMAs in US; second, expenditure in the rest of the world increases, thus raising labor demand for exports. The net effect of this change is to increase labor demand on net, and this effect shows up as a joint increase in manufacturing employment and wages across PUMAs.[36] Non-manufacturing employment decreases

[35] One may wonder whether there is at least a positive relation between dw_j^{NC} and dw_j: if that were the case, we could at least hope that the bias in the prediction of the impact of trade is constant across locations. Figure 22 in the Appendix shows that while there is a positive relation between dw_j^{NC} and dw_j in locations where CEP is active, the realized change in wage dw_j has basically no relation with dw_j^{NC} in locations where CEP is inactive.

[36] In models with more than two countries, the impact of such a change could be more elaborate. For example, the increase in expenditure in a foreign country may benefit more third countries than US, to the point of reducing labor demand in manufacturing

Figure 13: Overpredictions in a No-Commuting Model



This figure shows the distribution of the overprediction of changes in wages generated by a model with no commuting, if the true data-generating process actually has commuting flows; in particular, the overprediction for location j is defined as the change in wage in the location in the no-commuting model divided by the change in the with-commuting model. The left and right panel refer to locations not active and active in CEP, respectively.

everywhere, as more labor force is drawn into manufacturing. For some locations, total employment will increase on balance; of course, as total employment in US has to stay constant, however, the net impact of the increase in productivity abroad will be to shrink employment in other locations.

An interesting aspect of this exercise is the impact across PUMAs on the average real wage of residents in a location (no matter where they work), and its relation with the change in the nominal wage of agents working in j, w_j (no matter where they live). Remember that the average real wage in a location r is given by the average nominal wage across all destinations reached from r, $\mathcal{J}(r)$, weighted by commuting shares, and divided by the price index in the PUMA. Typically, the empirical literature focuses on wages paid to workers in a location, while for welfare analysis the real wage is more relevant.

There are three main factors which distinguish the impact of any change on wages of workers vs. real wages of residents. First, for given composition of commuting flows, changes in the nominal wages across destinations $\mathcal{J}(r)$ may either reinforce or compensate each other. Second, changes in nominal wages in $\mathcal{J}(r)$ imply a reshuffling of commuting flows always towards job destinations where wages increase the most (or fall

and nominal wages overall. This is indeed one of the facts that allows Autor, Dorn, and Hanson (2013) to focus mainly - but not exclusively - on the consequences of the increase in import penetration of China in US. The importance of third-countries effects is discussed for example in Di Giovanni, Levchenko and Zhang (2013). The extension of this model to many countries is certainly relevant for policy analysis but is left for future work.

Table 13: Distributional impacts of a 3x increase in RoW manufacturing productivity

	Min	p10	p25	p50	p75	p90	Max	Mean
w. wkrs.	0.900	0.986	0.993	0.998	1.00	1.01	1.13	0.999
mfg. empl.	7.92	8.31	8.35	8.38	8.42	8.47	8.75	8.38
non-mfg. empl.	-9.94	-3.70	-2.56	-1.72	-1.01	-0.62	-0.027	-2.03
tot. empl.	-0.136	-0.012	-0.004	0.001	0.007	0.014	0.100	0.000
w. res.	0.206	0.216	0.218	0.219	0.220	0.222	0.242	0.219

This table reports selected percentiles in the distribution of the impact of changes in the productivity growth in the rest of the world across across PUMAs in US. Each line reports the percentage points change in a PUMA-level outcome from the initial steady state in 2005 to the new, counterfactual steady state. The first line refers to the average wage of agents working in a PUMA, the second, third and fourth to the employment in manufacturing and non-manufacturing sector ,and total employment, respectively, and the fifth to the average real wage of residents in a PUMA, computed as the average nominal wage across all destinations reached, weighted by commuting shares, divided by the price index in the PUMA.

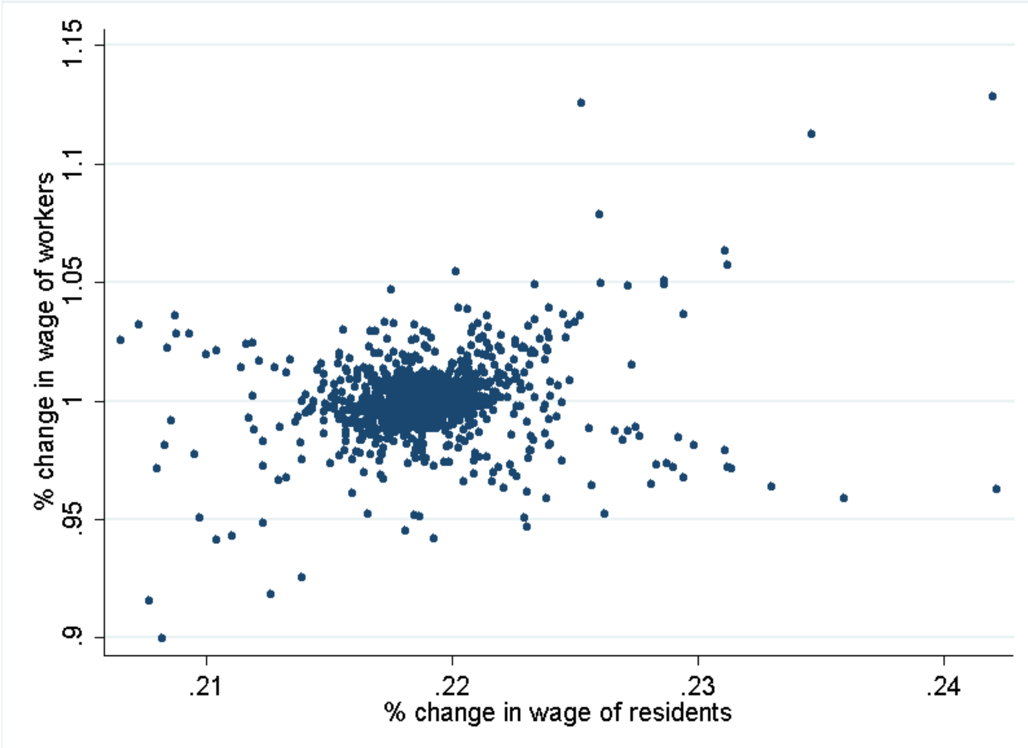
the least). Third, the change in wages paid to workers in r impacts the price of the non-tradable sector locally consumed, and always dampens the change in real wages there: for example, if w_r goes down, residents of r who do not commute outside have a lower nominal wage, but all residents have a lower cost of non-tradeables (and vice-versa if w_r goes up). The first two points become increasingly important as the available geographic unit of analysis becomes smaller, while the third applies even when areas are large and measured commuting ties are small.

We can use the simulation to investigate, for realistic parameter values, the impact on real wages of residents and their relation with the impact on the measured wages of workers, which would be the typical data available if one has no access to changes in prices of non-tradeable goods across residence locations and commuting ties among residence and job locations. The simulation suggests that the net effect of the productivity growth is about 0.21% on average across PUMAs, with the min/max spanning of one-fifth of this average change. The average change in the real wage is about five times smaller than the change in the nominal wage. Interestingly, Figure 14 shows that there is basically no correlation between changes in the nominal and real wages in the model, indicating a large impact of the confounding factors discussed above: a regression of the percentage change in $\tilde{w}_r/p_r$ on the percentage change in w_j produces a very flat slope of 0.04, and a R^2 of 4.4%. A focus on wages of workers would in this case reveal very limited information about the real geographic impact of the increase in the rest of the world's productivity.

9 Conclusion

In this paper, I develop an open economy model capable of studying the geographic incidence of episodes of international integration in general equilibrium. I investigate at theoretical level how trade shocks unfold through the territory of a country, focusing on two aspects, commuting ties among residence and job locations, and overlaps in the sectoral specialization in production. I have shown that the impact on wages of a change in an exogenous parameter of the model can be decomposed exactly in interpretable components, and that these components can also be measured in the data. The model can be estimated with publicly available data. While the framework can be used to study the consequences of immigration from third countries or changes in technology in a location within a country, I focus my analysis on the consequences of international integration,

Figure 14: Change in wages of workers and real wages of residents



This figure shows the relation between the change in nominal wages of workers in a PUMA, irrespective of their residence, vs. the change in the real wage of residents of a PUMA, irrespective of where they work, across the 1,230 PUMAs in US, following a 3 x increase in the productivity of the manufacturing sector of the rest of the world. The average real wage in a PUMA r is computed as the average nominal wage across all destinations reached by residents of r, weighted by commuting shares, and divided by the price index in the PUMA.

and perform two counterfactual exercises to illustrate a number of points on the geographic incidence of trade shocks. First, sector-level changes in trade frictions (or trade policy) can have an impact also on locations which are inactive in those sectors, simply because in equilibrium wages are changing in other, possibly closeby, locations. Second, if we were to study the consequences of trade in a model which features no commuting, we would over-predict the change in wages in locations which are active in the sector, and under-predict the effect in locations which are inactive in the sector. Third, the existence of commuting ties clarifies the conceptual difference between wages of agents working in a location (which are typically measured) and wages of residents in a location (which are usually more difficult to measure as they require data on the price of local, non-traded services). Fourth, this conceptual difference can translate in a very weak correlation between the impact of international integration on wages of workers vs. real wages of residents, and this suggests caution when interpreting the impact on wages of workers from a welfare perspective.

A Tables and Figures

A.1 Tables

Table 14: Number of destinations by PUMA, weighted by hours supplied

	Min	p10	p25	p50	p75	p90	Max	Mean	N
2000	17	33	44	61	101	174	257	81.54	236.6
2005	5	15	21	30	51	85	120	40.14	251.1
2006	6	15	21	31	52	88	121	40.94	261.0
2007	5	16	21	31	54	91	120	41.56	263.4
ALL	5	17	23	37	63	101	257	50.39	1,012.0

This table reports selected percentiles in the out-degree distribution of commuting patterns, i.e. the number of PUMAs of work chosen by at least one worker from any PUMA of residence. The first row uses Census 2000, the following three ACS in the reported years, and the last aggregates all years. Each PUMA of residence receives a weight proportional to the yearly hours of labor supplied by all residents. The last column reports total hours worked in a year, in billions.

Table 15: GMM estimates of eq. 21, with time dummies for the constant

	All distances		Truncated distances	
	Only 2005-07	All years	Only 2005-07	All years
η_0	4.46***	3.73***	4.84	4.65
	(1.30)	(0.83)	(3.70)	(3.47)
η_1	-0.41***	-0.42***	-0.71**	-0.72***
	(0.07)	(0.07)	(0.29)	(0.26)
η_2	0.21***	0.22***	0.40***	0.40***
	(0.04)	(0.04)	(0.15)	(0.14)
γ_1	-2.01*	-1.35***	-0.37	-0.28
	(1.05)	(0.50)	(0.38)	(0.29)
γ_2	0.26**	0.20***	0.04	0.04
	(0.13)	(0.07)	(0.04)	(0.03)
γ_3	-0.20*	-0.15***	-0.03	-0.03
	(0.10)	(0.06)	(0.03)	(0.03)
α_0	6.24**	4.64***	1.84*	1.64**
	(2.73)	(1.31)	(1.07)	(0.80)
α_1	0.04	0.03	0.01	0.01
	(0.03)	(0.02)	(0.01)	(0.01)
α_2	0.05	0.04*	0.01	0.01
	(0.03)	(0.02)	(0.01)	(0.01)
N	97,018	161,926	65,237	97,780

This table shows GMM estimates for eq. 21, adding year dummies to the constant. Each column reports the results of a particular GMM estimate: columns differ only according to the sample used. The first three columns use all observed bilateral commuting flows; the last three only bilateral commuting flows between PUMAs whose centroids are less than 137 miles away from each other (i.e., exclude the top 1% of commuters). Within each set of three columns, the first uses only commuting data from then Census 2000, the second only data from ACS 2005-2007, and the third all data together. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$. Standard errors in parentheses.

A.2 Figures

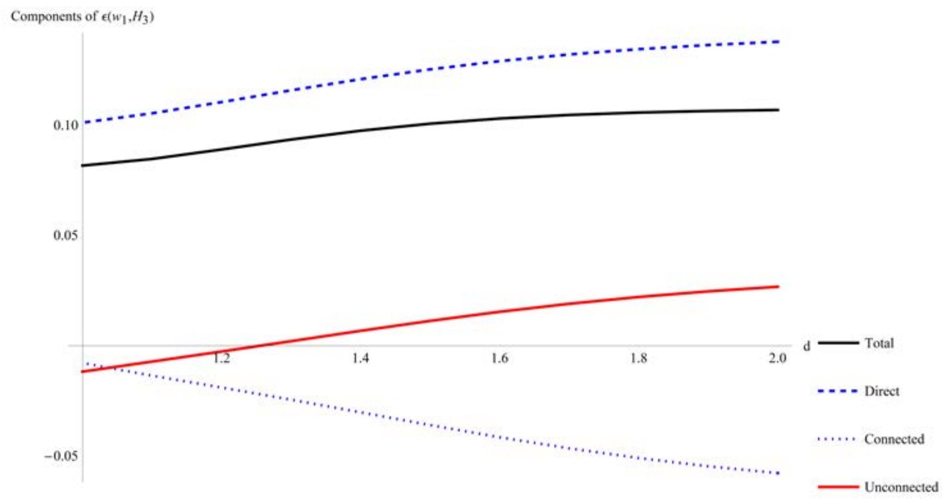


Figure 15: The equilibrium response of w_1 to H_3, and its components.

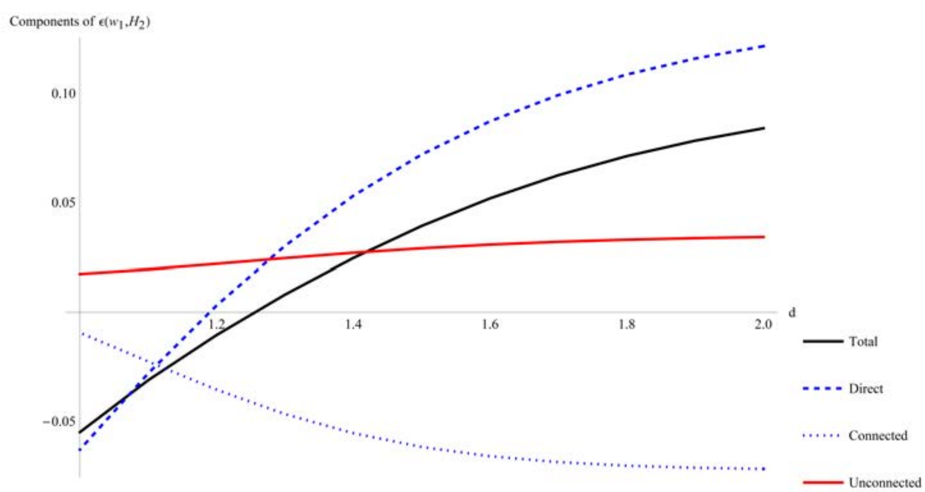


Figure 16: The equilibrium response of w_1 to H_2, and its components.

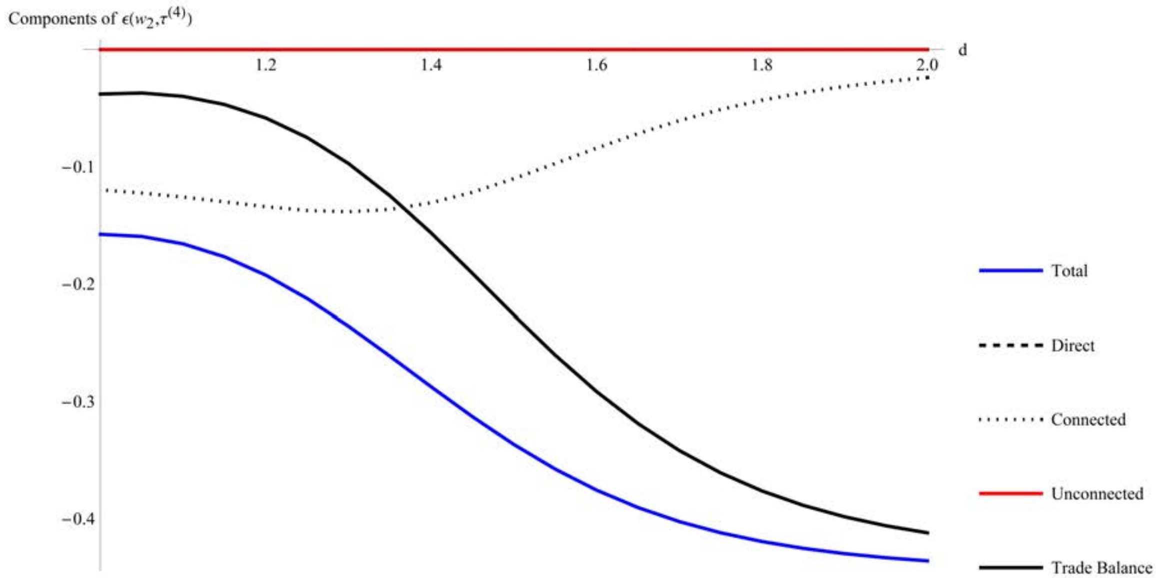


Figure 17: The equilibrium response of w_2 to $\tau^{(4)}$, and its components.

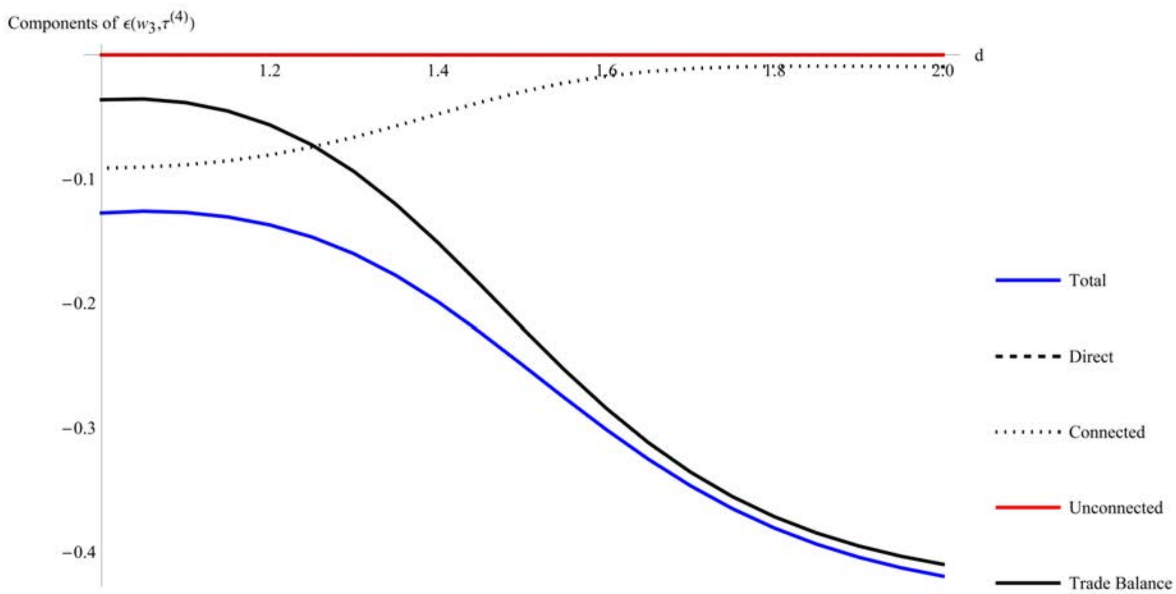


Figure 18: The equilibrium response of w_3 to $\tau^{(4)}$, and its components.

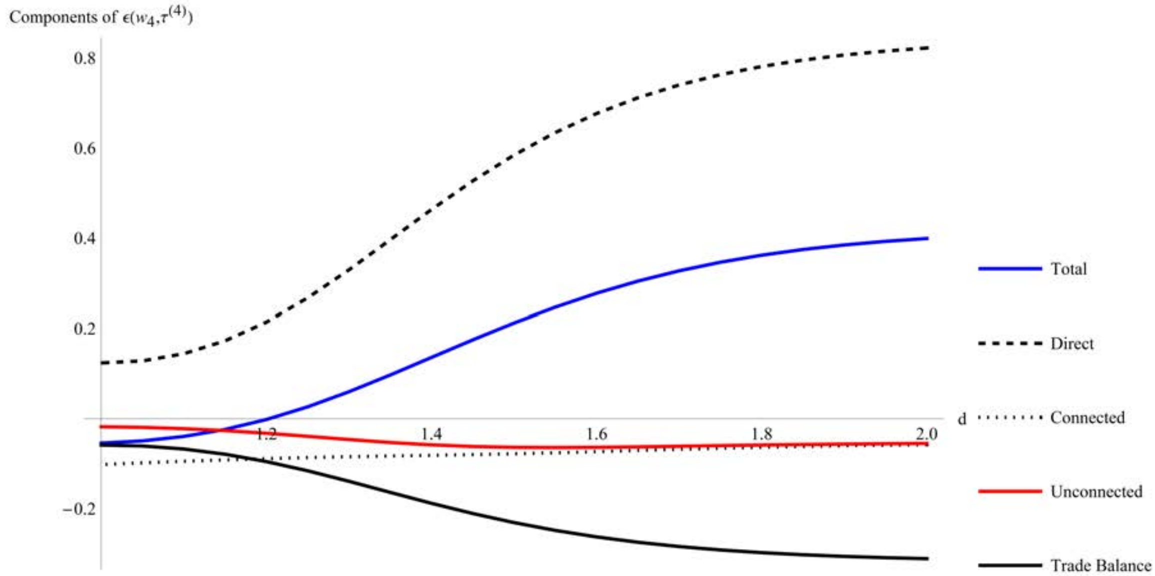


Figure 19: The equilibrium response of w_4 to $\tau^{(4)}$, and its components.

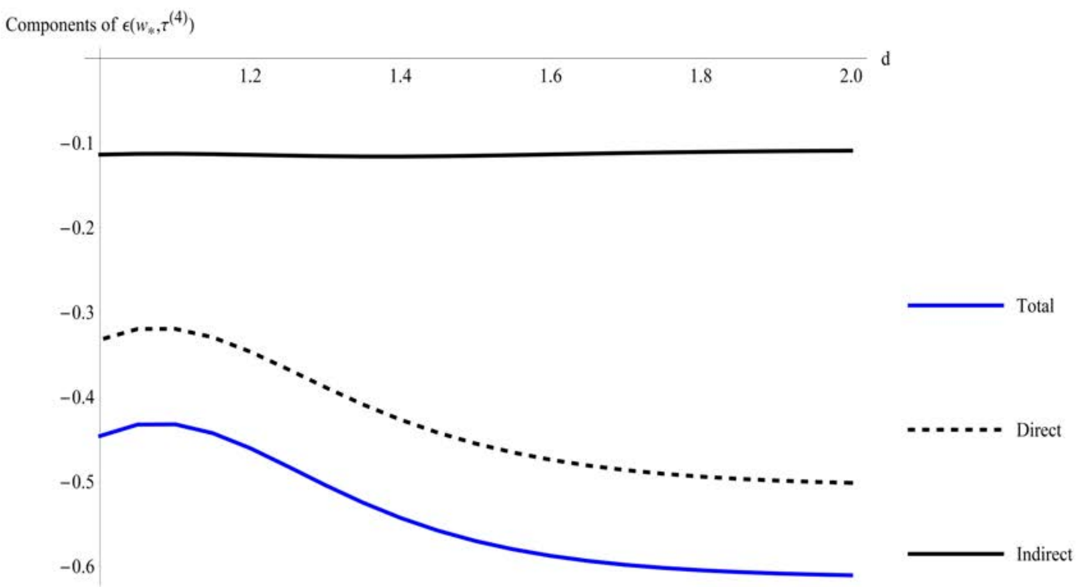
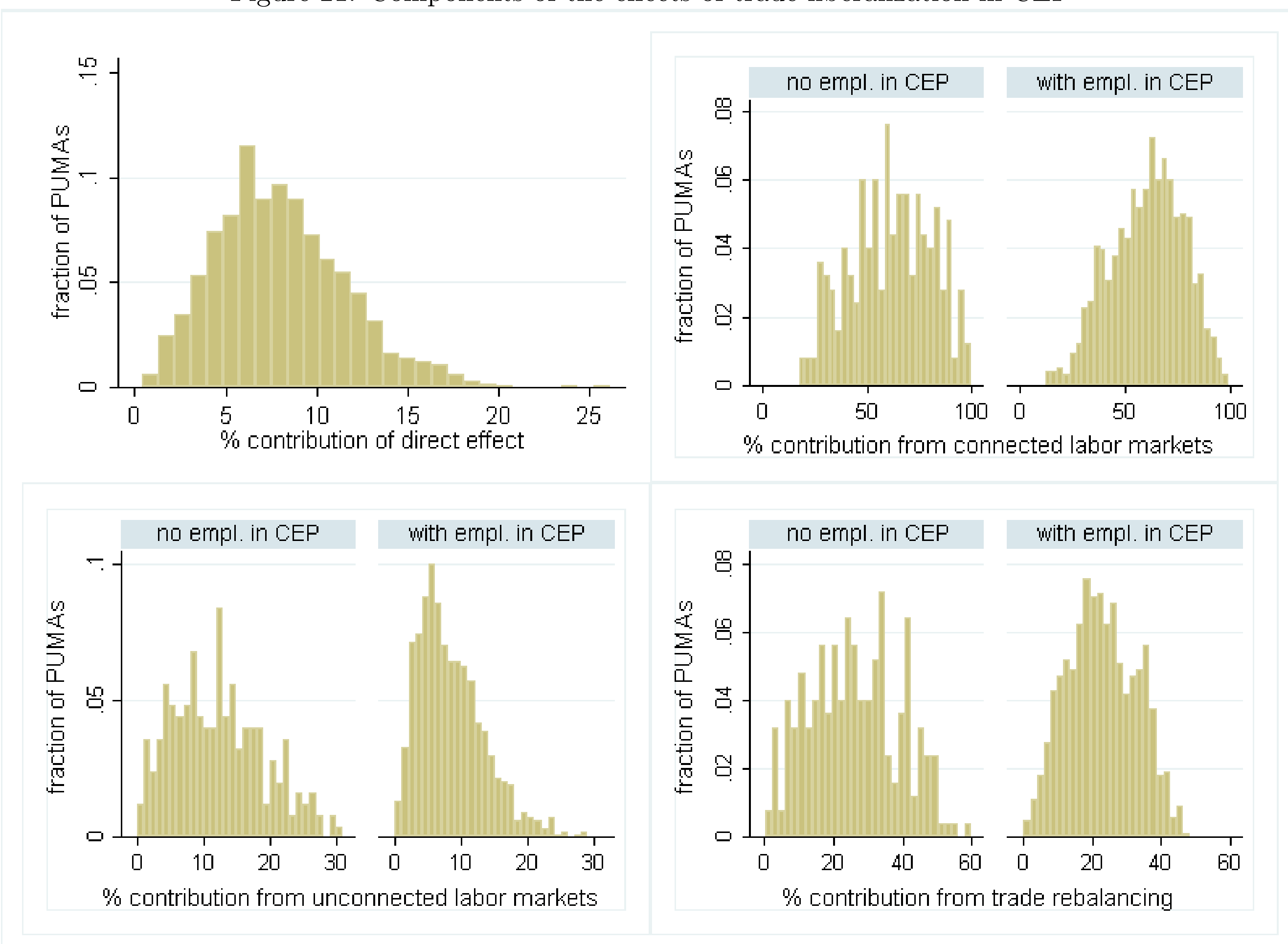


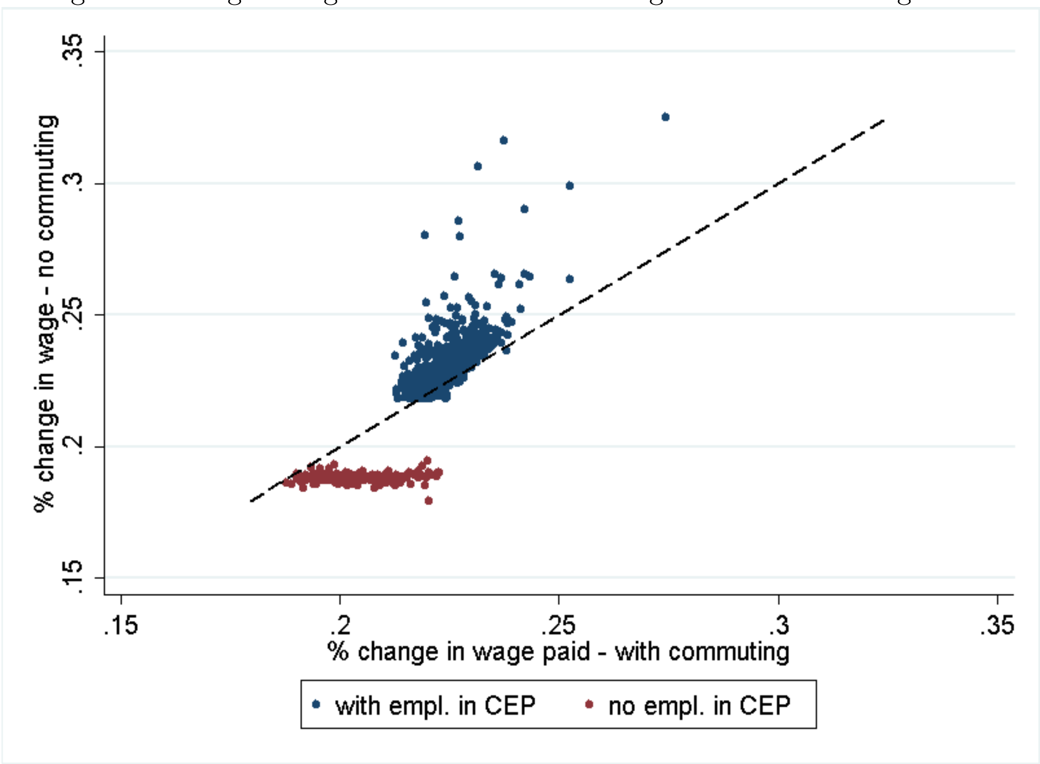
Figure 20: The equilibrium response of w_* to $\tau^{(4)}$, and its components.

Figure 21: Components of the effects of trade liberalization in CEP



This figure shows the contribution of different terms in Prop. 5 to the total impact of a liberalization in CEP, across PUMAs. The label along the x-axis of each panel reports the term to which the distribution is referring. For the direct effect only PUMAs active in CEP are considered; for the other three effects, the histograms are broken down according to whehter PUMAs have a positive empolyment in CEP or not.

Figure 22: Wage changes in the with-commuting and no-commuting model



This figure shows a scatterplot of the percentage change in wages paid to workers in a PUMA implied by the true, with-commuting model, vs. the percentage change in wages paid to workers in the same PUMA generated by a no-commuting model, when trade frictions in CEP are eliminated. The scatterplot separates locations where CEP is active from those where CEP is not active. The black, dashed line is a 45 degrees line.

B Proofs and Computations

B.1 Closed Economy

B.1.1 Elasticity of national income to a wage

The derivative of national income with respect to $w_{j'}$ is given by

$$\begin{aligned}
\frac{\partial X}{\partial w_{j'}} &= \sum_{r\in\mathcal{R}} H_r \frac{\partial}{\partial w_{j'}} \sum_{j\in\mathcal{J}(r)} m_{rj}w_j = \frac{\partial}{\partial w_{j'}}\left[\sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} H_r \sum_{j\in\mathcal{J}(r)} m_{rj}w_j + \sum_{r\in\mathcal{R}:j'\notin\mathcal{J}(r)} H_r \sum_{j\in\mathcal{J}(r)} m_{rj}w_j\right] = \\
&= \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} H_r \frac{\partial}{\partial w_{j'}} \sum_{j\in\mathcal{J}(r)} m_{rj}w_j + 0 = \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} H_r \frac{\partial}{\partial w_{j'}}\left(m_{rj'}w_{j'} + \sum_{j\in\mathcal{J}(r),j\neq j'} m_{rj}w_j\right) = \\
&= \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} H_r \left(m_{rj'} + w_{j'}\frac{\partial m_{rj'}}{\partial w_{j'}} + \sum_{j\in\mathcal{J}(r),j\neq j'} w_j \frac{\partial m_{rj}}{\partial w_{j'}}\right)
\end{aligned}$$

In terms of elasticities,

$$\begin{aligned}
\frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}} &= \frac{w_{j'}}{X}\sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} H_r \left(m_{rj'} + w_{j'}\frac{\partial m_{rj'}}{\partial w_{j'}} + \sum_{j\in\mathcal{J}(r),j\neq j'} w_j \frac{\partial m_{rj}}{\partial w_{j'}}\right) = \\
&= \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} \frac{H_r}{X}\left(w_{j'}m_{rj'} + w_{j'}m_{rj'}\frac{w_{j'}}{m_{rj'}}\frac{\partial m_{rj'}}{\partial w_{j'}} + \sum_{j\in\mathcal{J}(r),j\neq j'} w_j m_{rj}\frac{w_{j'}}{m_{rj}}\frac{\partial m_{rj}}{\partial w_{j'}}\right)
\end{aligned}$$

Using the formulas for elasticities given in (4),

$$\begin{aligned}
\frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}} &= \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} \frac{H_r}{X}\left(w_{j'}m_{rj'} + \frac{1}{\mu_r}w_{j'}m_{rj'}\left(1-m_{rj'}\right) - m_{rj'}\sum_{j\in\mathcal{J}(r),j\neq j'}\frac{1}{\mu_r}w_j m_{rj}\right) = \\
&= \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} \frac{H_r}{X}\left(w_{j'}m_{rj'} + \frac{1}{\mu_r}w_{j'}m_{rj'} - m_{rj'}\frac{1}{\mu_r}\sum_{j\in\mathcal{J}(r)} w_j m_{rj}\right)
\end{aligned}$$

Defining $\tilde{w}_r = \sum_{j\in\mathcal{J}(r)} m_{rj}w_j$ as the average nominal wage received by residents in r, $\iota_r \equiv H_r\tilde{w}_r/X$ as the share of r in national income, and $\iota_{rj'} = w_{j'}m_{rj'}/\tilde{w}_r$ as the share of income in r received from work performed in j,

$$\begin{aligned}
\frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}} &= \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} \frac{H_r\tilde{w}_r}{X}\left(\frac{w_{j'}m_{rj'}}{\tilde{w}_r} + \frac{1}{\mu_r}\frac{w_{j'}m_{rj'}}{\tilde{w}_r} - m_{rj'}\frac{1}{\mu_r}\sum_{j\in\mathcal{J}(r)}\frac{w_j m_{rj}}{\tilde{w}_r}\right) = \\
&= \sum_{r\in\mathcal{R}:j'\in\mathcal{J}(r)} \iota_r\left(\frac{1+\mu_r}{\mu_r}\iota_{rj'} - m_{rj'}\frac{1}{\mu_r}\right)
\end{aligned}$$

B.1.2 Existence and local uniqueness in closed economy

Proposition 1. *An equilibrium vector w^* exists.*

Proof. Consider the excess demand system

$$Z_j(w_1,...,w_R) = D(j;w) - L(j;w) = \frac{1}{w_j}\pi_j(w)\sum_{r\in\mathcal{R}} H_r \sum_{j'\in\mathcal{J}(r)} m_{rj'}(w)\,w_{j'} - \sum_{r\in\mathcal{J}^{-1}(j)} m_{rj}(w)\,H_r$$

for $j = 1,...,R$. Z is continuous as each Z_j is given by sums and products of continuous functions. We have made the dependence on w explicit now.

Labor supply is homogeneous of degree 0 ($HD0$) in wages, as $m_{rj}(w)$ is $HD0$. It is easy to see by inspection that labor demand is $HD0$ as well, since $\pi_j(w)$ is also $HD0$. Hence, Z is $HD0$.

Walras' law holds, since

$$\begin{aligned} w\cdot Z &= \sum_{j=1}^{R} w_j Z_j(w_1,...,w_R) = \sum_{j=1}^{R} w_j\left[\frac{1}{w_j}\pi_j(w)\sum_{r\in\mathcal{R}} H_r \sum_{j'\in\mathcal{J}(r)} m_{rj'}(w)\,w_{j'} - \sum_{r\in\mathcal{J}^{-1}(j)} m_{rj}(w)\,H_t\right] = \\ &= \sum_{j=1}^{R}\left[\pi_j(w)\sum_{r\in\mathcal{R}} H_r \sum_{j'\in\mathcal{J}(r)} m_{rj'}(w)\,w_{j'}\right] - \sum_{j=1}^{R}\left[w_j \sum_{r\in\mathcal{J}^{-1}(j)} m_{rj}(w)\,H_r\right] \end{aligned}$$

Using $m_r(j) = 0$ if $j \notin \mathcal{J}(r)$

$$\begin{aligned} w\cdot Z &= \sum_{j=1}^{R}\pi_j(w)\sum_{r=1}^{R} H_r \sum_{j'=1}^{R} m_{rj'}(w)\,w_{j'} - \sum_{j=1}^{R} w_j \sum_{r=1}^{R} m_{rj}(w)\,H_r = \\ &= \sum_{j=1}^{R}\pi_j(w)\sum_{r=1}^{R}\sum_{j'=1}^{R} w_{j'}m_{rj'}(w)\,H_r - \sum_{r=1}^{R}\sum_{j=1}^{R} w_j m_{rj}(w)\,H_r = \\ &= \left(\sum_{r=1}^{R}\sum_{j'=1}^{R} w_{j'}m_{rj'}(w)\,H_r\right)\left(\sum_{j=1}^{R}\pi_j(w)\right) - \sum_{r=1}^{R}\sum_{j=1}^{R} w_j m_{rj}(w)\,H_r = 0 \end{aligned}$$

as $\sum_{r=1}^{R}\sum_{j'=1}^{R} w_{j'}m_{rj'}(w)\,H_r$ is the national income, which does not depend on j, and π_j's must sum to 1.

Labor demand in each location j is always strictly positive, and has a lower bound at zero. Labor supply in each location j has an upper bound in $\sum_{r\in\mathcal{J}^{-1}(j)} H_r$. Let $k \equiv \max_j\left\{\sum_{r\in\mathcal{J}^{-1}(j)} H_r\right\}$. Hence, $Z_j(w) = D(j;w) - L(j;w) > -L(j;w) > -k$, $\forall w \in R^J_{++}$.

Consider now a sequence $\{w\}^n \to w^0$, where $w_j \neq 0$ and $w^0_{j'} = 0$ for some j'. Define $J' \subset \mathcal{R}$ as the set of $j' : w^0_{j'} = 0$. Suppose first that J' has cardinality 1, i.e. there is only one j' such that $w_{j'} = 0$: then $\pi_{j'}(w) \to 1$, and $m_{rj'}(w) \to 0$, while $\pi_j(w) \to \pi_j\left(w^0\right) = 0$, and $m_{rj}(w) \to m_{rj'}\left(w^0\right) \in (0,1]$, for $j \neq j'$. Hence, national income $\sum_{r\in\mathcal{R}} H_r \sum_{j\in\mathcal{J}(r)} m_{rj}(w)\,w_j$ approaches a positive constant, while

$$D\left(j';w\right) = \frac{1}{w_{j'}}\pi_{j'}(w)\sum_{r\in\mathcal{R}} H_r \sum_{j\in\mathcal{J}(r)} m_{rj}(w)\,w_j \to \frac{1}{w_{j'}}\sum_{r\in\mathcal{R}} H_r \sum_{j\in\mathcal{J}(r)} m_{rj}(w)\,w_j \to +\infty$$

Hence, the excess demand function $Z_j(w)$ also approaches $+\infty$ since labor supply in any location is always bounded above by k. Suppose now J' has cardinality larger than one. If, as $\{w\}^n \to w^0$, there is only one j' which goes to zero faster than all the others, then we can apply to that j' the argument above. If a number n of them approach zero at the same speed (i.e., their ratio approaches 1), then $\pi_{j'}(w) = 1/n$ for all of them, while labor supply always approaches a positive constant; we can apply then conclude that $D(j'; w) \to \infty$ for all those j'.

Hence, properties (i)-(v) in Proposition 17.B.2 of MWG hold. An equilibrium where $Z(w) = 0$ then exists by proposition 17.C.1 of MWG. ■

To prove Proposition 2 on local uniqueness, we first prove the following lemma.

Lemma 6 *Let $dZ_H(w) = \left[\frac{\partial Z_j}{\partial H_r}\right]_{r,j=1,...J-1}$ be a matrix where the $r, j-th$ element is the derivative of the excess labor demand in location j with respect to the population resident in r, H_r, evaluated at the wage vector w. $dZ_H(w)$ is generically invertible.*

Proof. Denote the $r, j-th$ element of $dZ_H(w)$ with $f_{rj}(w)$. From the expression of the excess labor demand, $f_{ij}(w) : R^J_{++} \to R$ and $f_{ij}(w) \in C^\infty$. The determinant of $dZ_H(w)$ is then a function $a(w) : R^J_{++} \to R$, and is also C^∞. We now show that the set $\left\{w \in R^J_{++} : a(w) = 0\right\}$ has zero Lebesgue measure in R^J_{++}. Note first that to show this property, it is sufficient to show that for a fixed arbitrary vector $w_{-1} \equiv [w_2, ..., w_J]$, there is no open interval for w_1 where $a(w_1; w_{-1}) = 0$ (i.e., there is no interval for w_1 where $a(w_1; w_{-1})$ is flat and equal to zero) ; in this case, in fact, there would be at most a countable set of points where $dZ_H(w)$ is not invertible, and this set would have measure zero. Fix then an arbitrary $w_{-1} \in R^{J-1}_{++}$, and consider the function $a(w_1; w_{-1})$, viewed as a function of one variable, w_1. Two cases are possible: either $\nexists\ w_1 \in R_{++} : a(w_1; w_{-1}) = 0$, or $\exists\ w_1 \in R_{++} : a(w_1; w_{-1}) = 0$. In the first case, the determinant is never zero, and hence $dZ_H(w_1; w_{-1})$ is invertible for all $\left\{w' \in R^J_{++} : w'_{-1} = w_{-1},\ w_1 \in R_{++}\right\}$. In the second case, the matrix is not invertible for at least some w_1; for this case, we show that $\exists\delta > 0$ such that $\forall\tilde{w} \in \{\tilde{w} \in R_{++} : |\tilde{w} - w_1| < \delta, \tilde{w} \neq w_1\}$, it is true that $a(\tilde{w}; w_{-1}) \neq 0$. Note in fact that the point $(w_1; w_{-1})$ can either be a critical point for $a(w)$ or not. If it is not, then $\partial a(\tilde{w}; w_{-1}) / \partial\tilde{w}|_{\tilde{w}=w_1} \neq 0$: hence $a(w_1; w_{-1}) \neq 0$ in a neighborhood of w_1. If it is a critical point, then $\partial a(\tilde{w}; w_{-1}) / \partial\tilde{w}|_{\tilde{w}=w_1} = 0$; however, since $a(w) \in C^\infty$, it is generically a Morse function, i.e., all critical points are non-degenerate, and hence, $\partial^2 a(\tilde{w}; w_{-1}) / \partial\tilde{w}^2|_{\tilde{w}=w_1} \neq 0$;*a fortiori*, there cannot be an interval where the second derivative is flat, i.e., where there is an interval of critical points around w_1. These arguments imply that for an arbitrary $w_{-1} \in R^{J-1}_{++}$, either the determinant of $dZ_H(w)$ is non-zero for all w_1, or it is zero for a countable set of values for w_1; in any case, they imply that $\left\{w \in R^J_{++} : a(w) = 0\right\}$ has dimension at most $J-1$, and hence it has zero Lebesgue measure in R^J_{++}. Hence, $a(w_1; w_{-1}) \neq 0$ generically, and $dZ_H(w)$ is generically invertible. ■

Proposition 2. *Let w^* be an equilibrium; then this equilibrium is generically locally unique.*

Proof. Note that $dZ_H(w^*)$ is generically invertible by Lemma 6. Hence, we can move $dZ_H(w^*)$ in any desired direction in R^{J-1} by setting dH vector appropriately. By the transversality theorem, since $dZ_{w,H}$ has generically rank $J-1$, the $(J-1) \times (J-1)$ matrix dZ_w also has rank $J-1$ generically. Hence, the equilibrium is generically locally unique. ■

B.1.3 Transmission of an immigration shock

Proposition 3. *The elasticity of the wage paid in any location j to the increase in the population in a location k is*

$$\varepsilon\left(w_j, H_k\right) = \underbrace{\iota_k \frac{1-\iota_{kj}/\pi_j}{\varepsilon\left(L\left(j\right), w_j\right) - \varepsilon\left(D\left(j\right), w_j\right)}}_{\text{direct effect}} + \underbrace{\sum_{j' \in \mathcal{J}^2(j)\setminus\{j\}} \frac{\varepsilon\left(D\left(j\right), w_{j'}\right) - \varepsilon\left(L\left(j\right), w_{j'}\right)}{\varepsilon\left(L\left(j\right), w_j\right) - \varepsilon\left(D\left(j\right), w_j\right)} \varepsilon\left(w_{j'}, H_k\right)}_{\text{indirect effect from connected labor markets}}$$
$$+ \underbrace{\sum_{j' \in \mathcal{R}\setminus\mathcal{J}^2(j)} \frac{\varepsilon\left(D\left(j\right), w_{j'}\right)}{\varepsilon\left(L\left(j\right), w_j\right) - \varepsilon\left(D\left(j\right), w_j\right)} \varepsilon\left(w_{j'}, H_k\right)}_{\text{indirect effects from unconnected labor markets}}$$

where $\iota_k \equiv H_k \tilde{w}_k / X$ is the share of national income accruing to residents in location k (having an average wage of $\tilde{w}_k$ overall), $\iota_{kj} \equiv m_{kj} w_j / \tilde{w}_k$ is the share of income of residents in k coming from work performed in j, and the right-hand side is evaluated at the equilibrium w.

Proof. We will totally differentiate the equilibrium condition $L(j) = D(j)$ with respect to H_k and the wage vector of the whole economy.

The total differential of the labor supply with respect to the wages in the whole economy only involves wages in the local labor market of j, $\mathcal{J}^2(j)$. this with respect to all the wages (with the exception of w_1) in the two-neighborhood of j and H_k

$$\frac{\partial L\left(j\right)}{\partial H_k} dH_k + \frac{\partial L\left(j\right)}{\partial w_j} dw_j + \sum_{j' \in \mathcal{J}^2(j)\setminus\{j\}} \frac{\partial L\left(j\right)}{\partial w_{j'}} dw_{j'}$$

Labor demand depends instead on the wages of the whole economy; its total differential is

$$\frac{\pi_j}{w_j} \frac{\partial X}{\partial H_k} dH_k + \frac{\partial D\left(j\right)}{\partial w_j} dw_j + \sum_{j' \in \mathcal{R}\setminus\{j\}} \frac{\partial D\left(j\right)}{\partial w_{j'}} dw_{j'}$$

Equating these two terms, collecting those referring to market j, and dividing through dH_k,

$$\frac{\partial L\left(j\right)}{\partial H_k} dH_k + \frac{\partial L\left(j\right)}{\partial w_j} dw_j + \sum_{j' \in \mathcal{J}^2(j)\setminus\{j\}} \frac{\partial L\left(j\right)}{\partial w_{j'}} dw_{j'} = \frac{\pi_j}{w_j} \frac{\partial X}{\partial H_k} dH_k + \frac{\partial D\left(j\right)}{\partial w_j} dw_j + \sum_{j' \in \mathcal{R}\setminus\{j\}} \frac{\partial D\left(j\right)}{\partial w_{j'}} dw_{j'}$$
$$\frac{\partial L\left(j\right)}{\partial w_j} \frac{dw_j}{dH_k} - \frac{\partial D\left(j\right)}{\partial w_j} \frac{dw_j}{dH_k} = \frac{\pi_j}{w_j} \frac{\partial X}{\partial H_k} - \frac{\partial L\left(j\right)}{\partial H_k} + \sum_{j' \in \mathcal{R}\setminus\{j\}} \left(\frac{\partial D\left(j\right)}{\partial w_{j'}} - \frac{\partial L\left(j\right)}{\partial w_{j'}}\right) \frac{dw_{j'}}{dH_k} + \sum_{j' \notin \mathcal{J}^2(j)} \frac{\partial D\left(j\right)}{\partial w_{j'}} \frac{dw_{j'}}{dH_k}$$

Multiplying both sides by $H_k / L(j)$, and recalling that in equilibrium $L(j) = D(j)$, we convert these terms into elasticities

$$\left(\varepsilon\left(L\left(j\right), w_j\right) - \varepsilon\left(D\left(j\right), w_j\right)\right) \varepsilon\left(w_j, H_k\right) = \frac{H_k}{L\left(j\right)} \frac{\pi_j}{w_j} \frac{\partial X}{\partial H_k} - \frac{H_k}{L\left(j\right)} \frac{\partial L\left(j\right)}{\partial H_k} +$$
$$+ \sum_{j' \in \mathcal{R}\setminus\{j\}} \left(\varepsilon\left(D\left(j\right), w_{j'}\right) - \varepsilon\left(L\left(j\right), w_{j'}\right)\right) \varepsilon\left(w'_j, H_k\right) + \sum_{j' \notin \mathcal{J}^2(j)} \varepsilon\left(D\left(j\right), w_{j'}\right) \varepsilon\left(w_{j'}, H_k\right)$$

In the first term, we note that $\frac{\partial X}{\partial H_k} = \tilde{w}_k$, $\frac{\partial L(j)}{\partial H_k} = m_{kj}$, $\pi_j / \left(L(j) w_j\right) = 1/X$; then, we collect $\frac{H_k}{X}\tilde{w}_k \equiv \iota_k$ out and use $\frac{m_k(j) w_j}{\tilde{w}_k} = \iota_{kj}$, to get

$$\frac{H_k}{L(j)} \frac{\pi_j}{w_j} \frac{\partial X}{\partial H_k} - \frac{H_k}{L(j)} \frac{\partial L(j)}{\partial H_k} = \frac{H_k}{X}\tilde{w}_k - \frac{H_k}{L(j)} m_{kj} = \iota_k \left(1 - \iota_{kj}/\pi_j\right)$$

We use this expression to substitute the first term, divide both sides through $\varepsilon\left(L(j), w_j\right) - \varepsilon\left(D(j), w_j\right)$, and obtain our result. ■

B.1.4 Response of the $\tilde{w}_r$ to changes in H_k

The wage of residents in location j responds as:

$$\frac{H_k}{\tilde{w}_r} \frac{\partial \tilde{w}_r}{\partial H_k} = \frac{H_k}{\tilde{w}_r} \frac{\partial}{\partial H_k} \sum_{j \in \mathcal{J}(r)} m_{rj} w_j = \sum_{j \in \mathcal{J}(r)} \frac{m_{rj} w_j}{\tilde{w}_r} \frac{H_k}{w_j} \frac{\partial w_j}{\partial H_k} + \sum_{j \in \mathcal{J}(r)} \frac{m_{rj} w_j}{\tilde{w}_r} \frac{H_k}{m_{rj}} \frac{\partial m_{rj}}{\partial H_k}$$

The elasticity of the commuting flow from r to j to changes in H_k is

$$\begin{aligned}
\frac{H_k}{m_{rj}} \frac{\partial m_{rj}}{\partial H_k} &= \frac{H_k}{m_{rj}} \frac{\left(w_j/d_{rj}\right)^{1/\mu_r} \mu_r^{-1} w_j^{-1} \frac{\partial w_j}{\partial H_k} \left[\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r}\right]}{\left[\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r}\right]^2} + \\
- & \quad \frac{H_k}{m_{rj}} \frac{\left(w_j/d_{rj}\right)^{1/\mu_r} \left[\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r} \mu_r^{-1} w_{j'} \frac{\partial w_j}{\partial H_k}\right]}{\left[\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r}\right]^2} = \\
&= \frac{1}{\mu_r} \frac{H_k w_j^{-1} \frac{\partial w_j}{\partial H_k} \left[\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r}\right] - \left[\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r} H_k w_{j'}^{-1} \frac{\partial w_j}{\partial H_k}\right]}{\left[\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r}\right]} = \\
&= \frac{1}{\mu_r} \left(\varepsilon\left(w_j, H_k\right) - \frac{\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r} \varepsilon\left(w_{j'}, H_k\right)}{\sum_{j' \in \mathcal{J}(r)} \left(w_{j'}/d_{rj'}\right)^{1/\mu_r}}\right) = \frac{1}{\mu_r} \left(\varepsilon\left(w_j, H_k\right) - \sum_{j' \in \mathcal{J}(r)} m_{rj'} \varepsilon\left(w_{j'}, H_k\right)\right)
\end{aligned}$$

Hence, we have

$$\begin{aligned}
\frac{H_k}{\tilde{w}_r}\frac{\partial \tilde{w}_r}{\partial H_k} &= \sum_{j\in\mathcal{J}(r)} \frac{m_{rj}w_j}{\tilde{w}_r}\varepsilon\left(w_j, H_k\right) + \sum_{j\in\mathcal{J}(r)} \frac{m_{rj}w_j}{\tilde{w}_r}\frac{1}{\mu_r}\left(\varepsilon\left(w_j, H_k\right) - \sum_{j'\in\mathcal{J}(r)} m_{rj'}\varepsilon\left(w_{j'}, H_k\right)\right) = \\
&= \sum_{j\in\mathcal{J}(r)} \frac{m_{rj}w_j}{\tilde{w}_r}\varepsilon\left(w_j, H_k\right) + \frac{1}{\mu_r}\sum_{j\in\mathcal{J}(r)} \frac{m_{rj}w_j}{\tilde{w}_r}\varepsilon\left(w_j, H_k\right) - \frac{1}{\mu_r}\sum_{j'\in\mathcal{J}(r)} m_{rj'}\varepsilon\left(w_{j'}, H_k\right)\sum_{j\in\mathcal{J}(r)}\frac{m_r\left(j\right)w_j}{\tilde{w}_r} = \\
&= \sum_{j\in\mathcal{J}(r)} \frac{m_{rj}w_j}{\tilde{w}_r}\varepsilon\left(w_j, H_k\right) + \frac{1}{\mu_r}\sum_{j\in\mathcal{J}(r)} \frac{m_{rj}w_j}{\tilde{w}_r}\varepsilon\left(w_j, H_k\right) - \frac{1}{\mu_r}\sum_{j'\in\mathcal{J}(r)} m_{rj'}\varepsilon\left(w_{j'}, H_k\right) = \\
&= \sum_{j\in\mathcal{J}(r)} \frac{m_{rj}w_j}{\tilde{w}_r}\varepsilon\left(w_j, H_k\right) + \frac{1}{\mu_r}\sum_{j\in\mathcal{J}(r)}\left(\frac{w_j}{\tilde{w}_r}-1\right) m_{rj}\varepsilon\left(w_j, H_k\right) = \\
&= \sum_{j\in\mathcal{J}(r)}\left[\frac{w_j}{\tilde{w}_r} + \frac{1}{\mu_r}\left(\frac{w_j}{\tilde{w}_r}-1\right)\right] m_{rj}\varepsilon\left(w_j, H_k\right)
\end{aligned}$$

B.2 Open Economy

B.2.1 Preliminaries

Many of the computations below will refer to the elasticity of trade shares with respect to wages. We report all these elasticities here for convenience.

Recall that

$$\begin{aligned}
\Phi^{(s)} &\equiv T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta} + \sum_{j\in\mathcal{R}} T_j^{(s)}w_j^{-\theta} \\
\Phi_*^{(s)} &\equiv T_*^{(s)}w^{*-\theta} + \sum_{j\in\mathcal{R}} T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}
\end{aligned}$$

and

$$\begin{aligned}
\pi_j^{(s)} &= T_j^{(s)}w_j^{-\theta}/\Phi^{(s)} \\
\pi_*^{(s)} &= T_*^{(s)}\left(\tau^{(s)}w^*\right)^{-\theta}/\Phi^{(s)} \\
\pi_{*j}^{(s)} &= T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}/\Phi_*^{(s)}
\end{aligned}$$

Then, $\pi_j^{(s)}$ has the following elasticities to wages and tariffs $\tau^{(s)}$:

$$\begin{aligned}
\frac{w_{j'}}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial w_{j'}} &= \theta \frac{w_{j'}}{\pi_j^{(s)}}\frac{T_j^{(s)}w_j^{-\theta}T_{j'}^{(s)}w_{j'}^{-\theta-1}}{\Phi^{(s)2}} = \theta\pi_{j'}^{(s)} \\
\frac{w_j}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial w_j^{(s)}} &= -\theta \frac{w_j}{\pi_j^{(s)}}\frac{T_j^{(s)}w_j^{-\theta-1}\Phi^{(s)} - T_j^{(s)}w_j^{-\theta}T_j^{(s)}w_j^{-\theta-1}}{\Phi^{(s)2}} = -\theta\left(1-\pi_j^{(s)}\right) \\
\frac{w_*}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial w_*} &= \theta \frac{w_*}{\pi_j^{(s)}}\frac{T_j^{(s)}w_j^{-\theta}T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}w_*^{-1}}{\Phi^{(s)2}} = \theta\pi_*^{(s)} \\
\frac{\tau^{(s)}}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial \tau^{(s)}} &= \theta \frac{\tau^{(s)}}{\pi_j^{(s)}}\frac{T_j^{(s)}w_j^{-\theta}T_*^{(s)}\tau^{(s)-\theta}w_*^{-\theta}\tau^{(s)-1}}{\Phi^{(s)2}} = \theta\pi_*^{(s)}
\end{aligned}$$

The share of expenditure on foreign goods by the home economy, $\pi_*^{(s)}$, has the following elasticities to wages and tariffs $\tau^{(s)}$:

$$\begin{aligned}
\frac{w_*}{\pi_*^{(s)}}\frac{\partial \pi_*^{(s)}}{\partial w_*} &= \frac{w_*}{\pi_*^{(s)}}\frac{-\theta T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}w_*^{-1}\Phi + \theta T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}w_*^{-1}}{\Phi^{(s)2}} = -\theta\left(1-\pi_*^{(s)}\right) \\
\frac{\tau^{(s)}}{\pi_*^{(s)}}\frac{\partial \pi_*^{(s)}}{\partial \tau^{(s)}} &= \frac{\tau}{\pi_*^{(s)}}\frac{-\theta T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}\tau^{(s)-1}\Phi + \theta T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}\tau^{-1}}{\Phi^{(s)2}} = -\theta\left(1-\pi_*^{(s)}\right) \\
\frac{w_j}{\pi_*^{(s)}}\frac{\partial \pi_*^{(s)}}{\partial w_j} &= \theta \frac{w_j}{\pi_*^{(s)}}\frac{T_*^{(s)}\left(\tau^{(s)}w^*\right)^{-\theta}T_j^{(s)}w_j^{-\theta-1}}{\Phi^{(s)2}} = \theta\pi_j^{(s)}
\end{aligned}$$

while the share of foreign's expenditure on each location's productions $\pi_{*j}^{(s)}$, has the following elasticities to wages and tariffs $\tau^{(s)}$

$$\begin{aligned}
\frac{w_*}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial w_*} &= \theta \frac{w_*}{\pi_{*j}^{(s)}}\frac{T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}T_*^{(s)}w_*^{-\theta}w_*^{-1}}{\Phi_*^{(s)2}} = \theta\pi_{**}^{(s)} \\
\frac{\tau^{(s)}}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial \tau^{(s)}} &= -\theta \frac{\tau^{(s)}}{\pi_{*j}^{(s)}}\frac{T_j^{(s)}\left(\tau w_j\right)^{-\theta}\tau^{(s)-1}\Phi_*^{(s)} - T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}\sum_{j\in\mathcal{R}}T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}\tau^{(s)-1}}{\Phi_*^{(s)2}} = -\theta\pi_{**}^{(s)} \\
\frac{w_{j'}}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial w_{j'}} &= \theta \frac{w_{j'}}{\pi_{*j}^{(s)}}\frac{T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}T_{j'}^{(s)}\left(\tau^{(s)}w_{j'}\right)^{-\theta}w_{j'}^{-1}}{\Phi_*^{(s)2}} = \theta\pi_{*j'}^{(s)} \\
\frac{w_j}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial w_j} &= \theta \frac{w_j}{\pi_{*j}^{(s)}}\frac{-T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}w_j^{-1}\Phi_*^{(s)} + T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}w_j^{-1}}{\Phi_*^{(s)2}} = -\theta\left(1-\pi_{*j}^{(s)}\right)
\end{aligned}$$

Finally, the share of foreign's expenditure on foreign goods has the following elasticities to wages:

$$
\begin{aligned}
\frac{w_j}{\pi_{**}^{(s)}}\frac{\partial \pi_{**}^{(s)}}{\partial w_j} &= \frac{w_j}{\pi_{**}^{(s)}}\theta\frac{T_*^{(s)} w_*^{-\theta} T_j^{(s)}\left(\tau^{(s)} w_j\right)^{-\theta} w_j^{-1}}{\Phi_*^{(s)2}} = \theta\pi_{*j}^{(s)} \\
\frac{w^*}{\pi_{**}^{(s)}}\frac{\partial \pi_{**}^{(s)}}{\partial w^*} &= -\theta\frac{w^*}{\pi_{**}^{(s)}}\frac{T_*^{(s)} w_*^{-\theta-1}\Phi^* - T_*^{(s)} w_*^{-\theta} T_*^{(s)} w_*^{-\theta-1}}{\Phi_*^{(s)2}} = -\theta\left(1-\pi_{**}^{(s)}\right) \\
\frac{\tau^{(s)}}{\pi_{**}^{(s)}}\frac{\partial \pi_{**}^{(s)}}{\partial \tau^{(s)}} &= \theta\frac{\tau^{(s)}}{\pi_{**}^{(s)}}\frac{T_*^{(s)} w_*^{-\theta}\sum_{j\in\mathcal{R}} T_j^{(s)}\left(\tau^{(s)} w_j\right)^{-\theta}\tau^{-1}}{\Phi_*^{(s)2}} = \theta\left(1-\pi_{**}^{(s)}\right)
\end{aligned}
$$

B.2.2 Elasticity of labor demand

Labor demand in j is given by

$$
D\left(j\right) = \frac{1}{w_j}\left[\sum_{s\in S_M(j)}\left(\pi_j^{(s)}\alpha^{(s)} X + \pi_{*j}^{(s)}\alpha^{(s)} X_*\right) + \alpha^{(\bar{s})}\tilde{w}_j H_j\right]
$$

, with $\tilde{w}_r = \sum_{j'\in\mathcal{J}(r)} m_r\left(j'\right) w_{j'}$.

Note that, using (4),

$$
\begin{aligned}
\frac{\partial \tilde{w}_r}{\partial w_{j'}} &= \sum_{j\in\mathcal{J}(r)}\frac{\partial m_{rj} w_j}{\partial w_{j'}} = \sum_{j\in\mathcal{J}(r)\backslash j'}\frac{\partial m_{rj}}{\partial w_{j'}} w_j + m_{rj'} + \frac{\partial m_{rj'}}{\partial w_{j'}} w_{j'} \Longrightarrow \\
\frac{w_{j'}}{\tilde{w}_r}\frac{\partial \tilde{w}_r}{\partial w_{j'}} &= \sum_{j\in\mathcal{J}(r)\backslash j'}\frac{m_{rj} w_j}{\tilde{w}_r}\frac{w_{j'}}{m_{rj}}\frac{\partial m_{rj}}{\partial w_{j'}} + \frac{w_{j'} m_{rj'}}{\tilde{w}_r} + \frac{w_{j'}}{m_{rj'}}\frac{\partial m_{rj'}}{\partial w_{j'}}\frac{m_{rj'} w_{j'}}{\tilde{w}_r} = \\
&= \sum_{j\in\mathcal{J}(r)\backslash j'} -\frac{1}{\mu_r}\frac{m_{rj} w_j}{\tilde{w}_r} m_{rj} + \frac{w_{j'} m_{rj'}}{\tilde{w}_r} + \frac{1}{\mu_r}\left(1-m_{rj'}\right)\frac{m_{rj'} w_{j'}}{\tilde{w}_r} = \\
&= -\frac{1}{\mu_r}\sum_{j\in\mathcal{J}(r)}\frac{m_{rj} w_j}{\tilde{w}_r} m_{rj'} + \left(1+\frac{1}{\mu_r}\right)\frac{m_{rj'} w_{j'}}{\tilde{w}_r}
\end{aligned}
$$

The response to a change in w_j is

$$\begin{aligned}\frac{\partial D(j)}{\partial w_j} &= -\frac{1}{w_j^2}D(j)\,w_j + \frac{1}{w_j}\left[\sum_{s\in S_M(j)}\left(\frac{\partial \pi_j^{(s)}}{\partial w_j}\alpha^{(s)}X + \frac{\partial X}{\partial w_j}\alpha^{(s)}\pi_j^{(s)} + \frac{\partial \pi_{*j}^{(s)}}{\partial w_j}\alpha^{(s)}X_*\right)\right. \\ + & \left.\alpha^{(\bar{s})}\frac{\partial \tilde{w}_j}{\partial w_j}H_j\right] \Longrightarrow \\ \frac{w_j}{D(j)}\frac{\partial D(j)}{\partial w_j} &= -1 + \frac{1}{w_j D(j)}\sum_{s\in S_M(j)}\left(\frac{w_j}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial w_j}\pi_j^{(s)}\alpha^{(s)}X + \frac{w_j}{X}\frac{\partial X}{\partial w_j}\alpha^{(s)}\pi_j^{(s)}X\,+\right. \\ &\left. + \frac{w_j}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial w_j}\alpha^{(s)}\pi_{*j}^{(s)}X_*\right) + \frac{1}{w_j D(j)}\frac{w_j}{\tilde{w}_j}\frac{\partial \tilde{w}_j}{\partial w_j}\alpha^{(\bar{s})}\tilde{w}_j H_j \\ &= -1 + \sum_{s\in S_M(j)}\left[\left(-\theta\left(1-\pi_j^{(s)}\right) + \frac{w_j}{X}\frac{\partial X}{\partial w_j}\right)\delta_j^{(s)} - \theta\left(1-\pi_{*j}^{(s)}\right)\delta_{*j}^{(s)}\right] + \\ &+ \frac{w_j}{\tilde{w}_j}\frac{\partial \tilde{w}_j}{\partial w_j}\delta_j^{(\bar{s})}\end{aligned}$$

where $\delta^{(s)} \equiv \pi_j^{(s)}\alpha^{(s)}X/\left(w_j D(j)\right)$, $\delta_{*j}^{(s)} \equiv \alpha^{(s)}\pi_{*j}^{(s)}X_*/\left(w_j D(j)\right)$, and $\delta_j^{(\bar{s})} \equiv \alpha^{(\bar{s})}\tilde{w}_j H_j/\left(w_j D(j)\right)$.

To compute the response of labor demand in j to a change in a wage in $w_{j'}$ with $j \neq j'$ and $j' \in \mathcal{J}(j)$, we have

$$\begin{aligned}\frac{\partial D(j)}{\partial w_{j'}} &= \frac{1}{w_j}\left(\sum_{s\in S_M(j)}\left(\alpha^{(s)}X\frac{\partial \pi_j^{(s)}}{\partial w_{j'}} + \pi_j^{(s)}\alpha^{(s)}\frac{\partial X}{\partial w_{j'}} + \frac{\partial \pi_{*j}^{(s)}}{\partial w_{j'}}\alpha^{(s)}X_*\right) + \alpha^{(\bar{s})}\frac{\partial \tilde{w}_j}{\partial w_{j'}}H_j\right) \Longrightarrow \\ \frac{w_{j'}}{D(j)}\frac{\partial D(j)}{\partial w_{j'}} &= \frac{1}{w_j}\frac{1}{D(j)}\sum_{s\in S_M(j)}\left(\pi_j^{(s)}\alpha^{(s)}X\frac{w_{j'}}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial w_{j'}} + \pi_j^{(s)}\alpha^{(s)}X\frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}} + \frac{w_{j'}}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial w_{j'}}\alpha^{(s)}\pi_{*j}^{(s)}X_*\right) + \\ &+ \frac{1}{w_j}\frac{1}{D(j)}\frac{w_{j'}}{\tilde{w}_j}\frac{\partial \tilde{w}_j}{\partial w_{j'}}\alpha^{(\bar{s})}\tilde{w}_j H_j\end{aligned}$$

and since $D(j)\,w_j$ is in equilibrium the total labor payments to workers working in location j

$$\begin{aligned}\frac{w_{j'}}{D(j)}\frac{\partial D(j)}{\partial w_{j'}} &= \sum_{s\in S_M(j)}\left[\left(\theta\pi_{j'}^{(s)} + \frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}}\right)\frac{\pi_j^{(s)}\alpha^{(s)}X}{D(j)\,w_j} + \frac{\alpha^{(s)}\pi_{*j}^{(s)}X_*}{D(j)\,w_j}\theta\pi_{*j'}^{(s)}\right] + \frac{w_{j'}}{\tilde{w}_j}\frac{\partial \tilde{w}_j}{\partial w_{j'}}\frac{\alpha^{(\bar{s})}\tilde{w}_j H_j}{D(j)\,w_j} = \\ &= \sum_{s\in S_M(j)}\left[\left(\theta\pi_{j'}^{(s)} + \frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}}\right)\delta_j^{(s)} + \theta\pi_{*j'}^{(s)}\delta_{*j}^{(s)}\right] + \frac{w_{j'}}{\tilde{w}_j}\frac{\partial \tilde{w}_j}{\partial w_{j'}}\delta_j^{(\bar{s})} = \\ &= \sum_{s\in S_M(j)\cap S_M(j')}\theta\left(\pi_{j'}^{(s)}\delta_j^{(s)} + \pi_{*j'}^{(s)}\delta_{*j}^{(s)}\right) + \frac{w_{j'}}{X}\frac{\partial X}{\partial w_{j'}}\sum_{s\in S_M(j)}\delta_j^{(s)} + \frac{w_{j'}}{\tilde{w}_j}\frac{\partial \tilde{w}_j}{\partial w_{j'}}\delta_j^{(\bar{s})}\end{aligned}$$

where the last equality follows from the fact that $\pi_j^{(s)}$ and $\pi_{*j'}^{(s)}$ are a function of $w_{j'}$ if and only if $s \in \mathcal{S}(j')$. The last term disappears if $j' \notin \mathcal{J}(j)$ as changes in $w_{j'}$ would not directly affect the average wage in location j.

The response to a change in the foreign wage is

$$
\begin{aligned}
\frac{\partial D(j)}{\partial w_*} &= \frac{1}{w_j}\sum_{s\in S_M(j)}\left(\alpha^{(s)}X\frac{\partial \pi_j^{(s)}}{\partial w_*}+\frac{\partial \pi_{*j}^{(s)}}{\partial w_*}\alpha^{(s)}X_*+\frac{\partial X_*}{\partial w_*}\alpha^{(s)}\pi_{*j}^{(s)}\right)\Longrightarrow \\
\frac{w_*}{D(j)}\frac{\partial D(j)}{\partial w_*} &= \frac{1}{w_j}\frac{1}{D(j)}\sum_{s\in S_M(j)}\left(\pi_j^{(s)}\alpha^{(s)}X\frac{w_*}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial w_*}+\frac{w_*}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial w_*}\alpha^{(s)}\pi_{*j}^{(s)}X_*+\frac{w_*}{X_*}\frac{\partial X_*}{\partial w_*}\alpha^{(s)}\pi_{*j}^{(s)}X_*\right) \\
\frac{w_*}{D(j)}\frac{\partial D(j)}{\partial w_*} &= \sum_{s\in S_M(j)}\left[\delta_j^{(s)}\theta\pi_*^{(s)}+\left(\theta\pi_{**}^{(s)}+1\right)\delta_{*j}^{(s)}\right]
\end{aligned}
$$

B.2.3 Transmission of a change in τ

Proposition 5. *The elasticity of the wage paid in any location j to a change in tariff in sector s, $\tau^{(s)}$, is*

$$
\begin{aligned}
\varepsilon\left(w_j,\tau^{(s)}\right) = {} & \underbrace{\frac{\varepsilon\left(D(j),\tau^{(s)}\right)}{\varepsilon(L(j),w_j)-\varepsilon(D(j),w_j)}}_{\text{direct effect}}+\underbrace{\sum_{j'\in\mathcal{J}^2(j)\setminus\{j\}}\frac{\varepsilon\left(D(j),w_{j'}\right)-\varepsilon\left(L(j),w_{j'}\right)}{\varepsilon(L(j),w_j)-\varepsilon(D(j),w_j)}\varepsilon\left(w_{j'},\tau^{(s)}\right)}_{\text{indirect effect from connected labor markets}} \\
& +\underbrace{\sum_{j'\in\mathcal{R}\setminus\mathcal{J}^2(j)}\frac{\varepsilon\left(D(j),w_{j'}\right)}{\varepsilon(L(j),w_j)-\varepsilon(D(j),w_j)}\varepsilon\left(w_{j'},\tau^{(s)}\right)}_{\text{indirect effect from unconnected labor markets}}+ \\
& +\underbrace{\frac{\varepsilon\left(D(j),w^*\right)}{\varepsilon(L(j),w_j)-\varepsilon(D(j),w_j)}\varepsilon\left(w_*,\tau^{(s)}\right)}_{\text{indirect effect from trade balance}}
\end{aligned}
\tag{25}
$$

for a wage in a domestic economy, and

$$
\frac{\tau^{(s)}}{w^*}\frac{dw_*}{d\tau^{(s)}}=\underbrace{\frac{\varepsilon\left(-NX(s),\tau^{(s)}\right)}{\sum_{s\in\mathcal{S}_M}\varepsilon\left(NX(s),w_*\right)}}_{\text{direct effect}}+\underbrace{\frac{\sum_{j\in\mathcal{R}}\left(\sum_{s\in\mathcal{S}_M}\varepsilon\left(-NX(s),w_j\right)\frac{\tau^{(s)}}{w_j}\frac{dw_j}{d\tau^{(s)}}\right)}{\sum_{s\in\mathcal{S}_M}\varepsilon\left(NX(s),w_*\right)}}_{\text{indirect effect}}
\tag{26}
$$

in the foreign economy. In these expressions,

$$
\begin{aligned}
\varepsilon\left(D(j),\tau^{(s)}\right) &= \begin{cases}\theta\pi_*^{(s)}\delta_j^{(s)}-\theta\pi_{**}^{(s)}\delta_{*j}^{(s)} & if\ s\in\mathcal{S}(j)\\ 0 & if\ \ s\notin\mathcal{S}(j)\end{cases}, \\
\delta_j^{(s)} &\equiv \pi_j^{(s)}\alpha^{(s)}X/\left(D(j)\,w_j\right) \\
\delta_{*j}^{(s)} &\equiv \pi_{*j}^{(s)}\alpha^{(s)}X_*/\left(D(j)\,w_j\right)
\end{aligned}
\tag{27}
$$

and $\varepsilon\left(-NX(s),\tau^{(s)}\right)$ is the elasticity of net imports of sector s to $\tau^{(s)}$, and similar definitions apply to the other terms.

Proof. The total differential of the labor market equilibrium in location j follows strictly the arguments in the proof of Proposition 3. The only difference is in the direct effect. The response to a change in trade costs in sector s is (if $s \in \mathcal{S}_M(j)$)

$$\begin{aligned}
\frac{\partial D(j)}{\partial \tau^{(s)}} &= \frac{1}{w_j}\frac{\partial}{\partial \tau^{(s)}}\left(\pi_j^{(s)}\alpha^{(s)}X + \pi_{*j}^{(s)}\alpha^{(s)}X^*\right) = \frac{1}{w_j}\left(\frac{\partial \pi_j^{(s)}}{\partial \tau^{(s)}}\alpha^{(s)}X + \frac{\partial \pi_{*j}^{(s)}}{\partial \tau^{(s)}}\alpha^{(s)}X^*\right) \Longrightarrow \\
\frac{\tau^{(s)}}{D(j)}\frac{\partial D(j)}{\partial \tau^{(s)}} &= \frac{\tau^{(s)}}{\pi_j^{(s)}}\frac{\partial \pi_j^{(s)}}{\partial \tau^{(s)}}\frac{\alpha^{(s)}\pi_j^{(s)}X}{D(j)\,w_j} + \frac{\tau^{(s)}}{\pi_{*j}^{(s)}}\frac{\partial \pi_{*j}^{(s)}}{\partial \tau^{(s)}}\frac{\alpha^{(s)}\pi_{*j}^{(s)}X^*}{D(j)\,w_j} = \\
&= \theta\pi_*^{(s)}\delta_j^{(s)} - \theta\pi_{**}^{(s)}\delta_{*j}^{(s)}
\end{aligned}$$

where $\delta_j^{(s)} \equiv \pi_j^{(s)}\alpha^{(s)}X/\left(D(j)\,w_j\right)$ and $\delta_{*j}^{(s)} \equiv \pi_{*j}^{(s)}\alpha^{(s)}X^*/\left(D(j)\,w_j\right)$.

The total differential will include the impact of a change in w_* on labor demand in j, written as:

$$\begin{aligned}
\frac{\tau^{(s)}}{w_j}\frac{\frac{\partial D(j)}{\partial w^*}}{\frac{\partial L(j)}{\partial w_j} - \frac{\partial D(j)}{\partial w_j}}\frac{dw^*}{d\tau^{(s)}} &= \frac{\frac{\partial D(j)}{\partial w^*}\frac{w^*}{D(j)}}{\frac{w_j}{L(j)}\frac{\partial L(j)}{\partial w_j} - \frac{w_j}{D(j)}\frac{\partial D(j)}{\partial w_j}}\frac{\tau^{(s)}}{w^*}\frac{dw^*}{d\tau^{(s)}} = \\
= \frac{\varepsilon\left(D(j), w^*\right)}{\varepsilon\left(L(j), w_j\right) - \varepsilon\left(D(j), w_j\right)}\varepsilon\left(w_*, \tau^{(s)}\right)
\end{aligned}$$

The elasticity of w_* to changes in $\tau^{(s)}$ is determined by the trade balance:

$$\sum_{s\in\mathcal{S}_M}\pi_*^{(s)}\alpha^{(s)}X = \sum_{j\in\mathcal{R}}\sum_{s\in\mathcal{S}_M(j)}\pi_{*j}^{(s)}\alpha^{(s)}X^* \Longleftrightarrow \sum_{s\in\mathcal{S}_M}\frac{T_*^{(s)}\left(\tau^{(s)}w_*\right)^{-\theta}}{\Phi^{(s)}}\alpha^{(s)}X = \sum_{j\in R}\sum_{s\in\mathcal{S}_M(j)}\frac{T_j^{(s)}\left(\tau^{(s)}w_j\right)^{-\theta}}{\Phi^{(s)*}}\alpha^{(s)}X^*$$

The total differential of the trade balance with respect to all the wages and $\tau^{(s)}$ is

$$\begin{aligned}
&\sum_{j\in\mathcal{R}}\left(\sum_{s\in\mathcal{S}_M}\frac{\partial}{\partial w_j}\left(\pi_*^{(s)}\alpha^{(s)}X\right)dw_j\right) + \sum_{s\in\mathcal{S}_M}\frac{\partial \pi_*^{(s)}}{\partial w_*}\alpha^{(s)}Xdw_* + \frac{\partial \pi_*^{(s)}}{\partial \tau^{(s)}}\alpha^{(s)}Xd\tau^{(s)} = \\
&= \sum_{j\in\mathcal{R}}\frac{\partial}{\partial w_j}\left(\sum_{j'\in\mathcal{R}}\sum_{s\in\mathcal{S}_M(j')}\pi_{*j'}^{(s)}\alpha^{(s)}X^*\right)dw_j + \sum_{j\in R}\sum_{s\in\mathcal{S}_M(j)}\frac{\partial \pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}dw_* + \sum_{j'\in R: s\in\mathcal{S}_M(j')}\frac{\partial \pi_{*j'}^{(s)}}{\partial \tau^{(s)}}\alpha^{(s)}X^*d\tau^{(s)}
\end{aligned}$$

Oh the left-hand side, the first term is the change in total imports of a sector s as w_j changes, summed for all sectors s and then all j: it then gives the change in total imports as wages anywhere in the home economy change. The second term gives the change in total imports of a sector s as the foreign wage increases (and hence it is always negative). The third term is the change in imports of sector s as a consequence of the change in $\tau^{(s)}$, and is always positive if $d\tau^{(s)}$ is negative.

On the right-hand side, the first term is the change in exports in a sector s where j' is active as w_j increases, summed across all active sectors in j', and then across all locations j': this is the impact of a change in w_j on all exports, which is then summed across all origin locations j. The second term is the impact on exports of s as w_* increases, and is always positive. The third term is the impact of the change in $\tau^{(s)}$ on exports of sector s for a location j, summed across all locations which are active in such sector.

Grouping terms,

$$\sum_{j\in R}\sum_{s\in\mathcal{S}_M(j)}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}dw_*-\sum_{s\in\mathcal{S}_M}\frac{\partial\pi_*^{(s)}}{\partial w_*}\alpha^{(s)}Xdw_*=\frac{\partial\pi_*^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}Xd\tau^{(s)}-\sum_{j'\in\mathcal{R}:s\in\mathcal{S}_M(j')}\frac{\partial\pi_{*j'}^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}X^*d\tau^{(s)}+$$

$$+\quad\sum_{j\in\mathcal{R}}\left(\sum_{s\in\mathcal{S}_M}\frac{\partial\pi_*^{(s)}\alpha^{(s)}X}{\partial w_j}dw_j\right)-\sum_{j\in\mathcal{R}}\frac{\partial}{\partial w_j}\left(\sum_{j'\in\mathcal{R}}\sum_{s\in\mathcal{S}_M(j')}\pi_{*j'}^{(s)}\alpha^{(s)}X^*\right)dw_j\Longrightarrow$$

$$\sum_{s\in\mathcal{S}_M}\left[\left(\sum_{j:s\in\mathcal{S}_M(j)}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}\right)-\frac{\partial\pi_*^{(s)}}{\partial w_*}\alpha^{(s)}X\right]dw_*=\left[\frac{\partial\pi_*^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}X-\sum_{j'\in\mathcal{R}:s\in\mathcal{S}_M(j')}\frac{\partial\pi_{*j'}^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}X^*\right]d\tau^{(s)}+$$

$$+\sum_{j\in\mathcal{R}}\left[\left(\sum_{s\in\mathcal{S}_M}\frac{\partial\pi_*^{(s)}\alpha^{(s)}X}{\partial w_j}\right)-\left(\sum_{j'\in\mathcal{R}}\sum_{s\in\mathcal{S}_M(j')}\frac{\partial\pi_{*j'}^{(s)}\alpha^{(s)}X^*}{\partial w_j}\right)\right]dw_j$$

On the left hand side, we have the change in net exports of each sector as a function of a change in w_*; on the right-hand side, the change in net imports in response to a change in $\tau^{(s)}$, plus the change in net imports following a change in all w_j. Dividing through by $d\tau^{(s)}$ and expressing in terms of elasticities,

$$\sum_{s\in\mathcal{S}_M}\left[\left(\sum_{j:s\in\mathcal{S}_M(j)}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X_*}{\partial w_*}\right)-\frac{\partial\pi_*^{(s)}}{\partial w_*}\alpha^{(s)}X\right]\frac{\tau^{(s)}}{w^*}\frac{dw_*}{d\tau^{(s)}}=\frac{\tau^{(s)}}{w^*}\left[\frac{\partial\pi_*^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}X-\sum_{j'\in\mathcal{R}:s\in\mathcal{S}_M(j')}\frac{\partial\pi_{*j'}^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}X_*\right]+$$

$$+\quad\frac{\tau^{(s)}}{w^*}\sum_{j\in\mathcal{R}}\left[\sum_{s\in\mathcal{S}_M}\left(\frac{\partial\pi_*^{(s)}\alpha^{(s)}X}{\partial w_j}-\sum_{j':s\in\mathcal{S}_M(j')}\frac{\partial\pi_{*j'}^{(s)}\alpha^{(s)}X_*}{\partial w_j}\right)\right]\frac{dw_j}{d\tau^{(s)}}\Longrightarrow$$

$$\frac{\tau^{(s)}}{w^*}\frac{dw_*}{d\tau^{(s)}}=\frac{\tau^{(s)}}{w^*}\frac{\left[\frac{\partial\pi_*^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}X-\sum_{j'\in\mathcal{R}:s\in\mathcal{S}_M(j')}\frac{\partial\pi_{*j'}^{(s)}}{\partial\tau^{(s)}}\alpha^{(s)}X_*\right]}{\sum_{s\in\mathcal{S}_M}\left[\left(\sum_{j:s\in\mathcal{S}_M(j)}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}\right)-\frac{\partial\pi_*^{(s)}}{\partial w_*}\alpha^{(s)}X\right]}+$$

$$+\frac{\tau^{(s)}}{w^*}\frac{\sum_{j\in\mathcal{R}}\left[\sum_{s\in\mathcal{S}_M}\left(\frac{\partial\pi_*^{(s)}\alpha^{(s)}X}{\partial w_j}-\sum_{j':s\in\mathcal{S}_M(j')}\frac{\partial\pi_{*j'}^{(s)}\alpha^{(s)}X_*}{\partial w_j}\right)\right]\frac{dw_j}{d\tau^{(s)}}}{\sum_{s\in\mathcal{S}_M}\left[\left(\sum_{j:s\in\mathcal{S}_M(j)}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}\right)-\frac{\partial\pi_*^{(s)}}{\partial w_*}\alpha^{(s)}X\right]}$$

$$\frac{\tau^{(s)}}{w^*}\frac{dw_*}{d\tau^{(s)}}=\frac{\left[\frac{\tau^{(s)}}{\pi_*^{(s)}}\frac{\partial\pi_*^{(s)}}{\partial\tau^{(s)}}\pi_*^{(s)}\alpha^{(s)}X-\sum_{j'\in\mathcal{R}:s\in\mathcal{S}_M(j')}\frac{\tau^{(s)}}{\pi_{*j'}^{(s)}}\frac{\partial\pi_{*j'}^{(s)}}{\partial\tau^{(s)}}\pi_{*j'}^{(s)}\alpha^{(s)}X_*\right]}{\sum_{s\in\mathcal{S}_M}\left[\left(\sum_{j:s\in\mathcal{S}_M(j)}\frac{w^*}{\pi_{*j}^{(s)}\alpha^{(s)}X^*}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}\pi_{*j}^{(s)}\alpha^{(s)}X_*\right)-\frac{w^*}{\pi_*^{(s)}}\frac{\partial\pi_*^{(s)}}{\partial w_*}\pi_*^{(s)}\alpha^{(s)}X\right]}+$$

$$+\frac{\sum_{j\in\mathcal{R}}\left[\sum_{s\in\mathcal{S}_M}\left(\frac{w_j}{\pi_*^{(s)}\alpha^{(s)}X}\frac{\partial\pi_*^{(s)}\alpha^{(s)}X}{\partial w_j}\pi_*^{(s)}\alpha^{(s)}X-\sum_{j':s\in\mathcal{S}_M(j')}\frac{w_j}{\pi_{*j'}^{(s)}\alpha^{(s)}X^*}\frac{\partial\pi_{*j'}^{(s)}\alpha^{(s)}X^*}{\partial w_j}\pi_{*j'}^{(s)}\alpha^{(s)}X_*\right)\right]\frac{\tau^{(s)}}{w_j}\frac{dw_j}{d\tau^{(s)}}}{\sum_{s\in\mathcal{S}_M}\left[\left(\sum_{j:s\in\mathcal{S}_M(j)}\frac{w^*}{\pi_{*j}^{(s)}\alpha^{(s)}X_*}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}\pi_{*j}^{(s)}\alpha^{(s)}X_*\right)-\frac{w^*}{\pi_*^{(s)}}\frac{\partial\pi_*^{(s)}}{\partial w_*}\pi_*^{(s)}\alpha^{(s)}X\right]}$$

which in compact form can be written as,

$$\frac{\tau^{(s)}}{w^*}\frac{dw_*}{d\tau^{(s)}} = \frac{\varepsilon\left(-NX\left(s\right),\tau^{(s)}\right)}{\sum_{s\in\mathcal{S}_M}\varepsilon\left(NX\left(s\right),w_*\right)} + \frac{\sum_{j\in\mathcal{R}}\left(\sum_{s\in\mathcal{S}_M}\varepsilon\left(-NX\left(s\right),w_j\right)\frac{\tau^{(s)}}{w_j}\frac{dw_j}{d\tau^{(s)}}\right)}{\sum_{s\in\mathcal{S}_M}\varepsilon\left(NX\left(s\right),w_*\right)}$$

■

We can also rewrite the elasticity of w_* more explicitly. The numerator of the first term is

$$\begin{aligned}
&\frac{\tau^{(s)}}{\pi_*^{(s)}}\frac{\partial\pi_*^{(s)}}{\partial\tau^{(s)}}\pi_*^{(s)}\alpha^{(s)}X - \sum_{j'\in\mathcal{R}:s\in\mathcal{S}_M(j')}\frac{\tau^{(s)}}{\pi_{*j'}^{(s)}}\frac{\partial\pi_{*j'}^{(s)}}{\partial\tau^{(s)}}\pi_{*j'}^{(s)}\alpha^{(s)}X^* = \\
= \;& -\theta\left(1-\pi_*^{(s)}\right)\pi_*^{(s)}\alpha^{(s)}X + \theta\pi_{**}^{(s)}\sum_{j'\in\mathcal{R}:s\in\mathcal{S}_M(j')}\pi_{*j'}^{(s)}\alpha^{(s)}X_* = \\
= \;& -\theta\left(1-\pi_*^{(s)}\right)\pi_*^{(s)}\alpha^{(s)}X + \theta\left(1-\pi_{**}^{(s)}\right)\pi_{**}^{(s)}\alpha^{(s)}X_*
\end{aligned}$$

The numerator in each of the second terms is

$$\begin{aligned}
&\frac{w_j}{\pi_*^{(s)}\alpha^{(s)}X}\frac{\partial\pi_*^{(s)}\alpha^{(s)}X}{\partial w_j}\pi_*^{(s)}\alpha^{(s)}X - \sum_{j':s\in\mathcal{S}_M(j')}\frac{w_j}{\pi_{*j'}^{(s)}\alpha^{(s)}X^*}\frac{\partial\pi_{*j'}^{(s)}\alpha^{(s)}X^*}{\partial w_j}\pi_{*j'}^{(s)}\alpha^{(s)}X^* = \\
= \;& \left(\theta\pi_j^{(s)} + \frac{w_j}{X}\frac{\partial X}{\partial w_j}\right)\pi_*^{(s)}\alpha^{(s)}X - \theta\pi_{*j}^{(s)}\sum_{j':s\in\mathcal{S}_M(j')}\pi_{*j'}^{(s)}\alpha^{(s)}X^* = \\
= \;& \left(\theta\pi_j^{(s)} + \frac{w_j}{X}\frac{\partial X}{\partial w_j}\right)\pi_*^{(s)}\alpha^{(s)}X - \theta\left(1-\pi_{**}^{(s)}\right)\pi_{*j}^{(s)}\alpha^{(s)}X^*
\end{aligned}$$

The denominator in each of the two terms contains

$$\begin{aligned}
&\left(\sum_{j:s\in\mathcal{S}_M(j)}\frac{w^*}{\pi_{*j}^{(s)}\alpha^{(s)}X^*}\frac{\partial\pi_{*j}^{(s)}\alpha^{(s)}X^*}{\partial w_*}\pi_{*j}^{(s)}\alpha^{(s)}X^*\right) - \frac{w^*}{\pi_*^{(s)}}\frac{\partial\pi_*^{(s)}}{\partial w_*}\pi_*^{(s)}\alpha^{(s)}X = \\
= \;& \left(\theta\pi_{**}^{(s)}+1\right)\sum_{j:s\in\mathcal{S}_M(j)}\pi_{*j}^{(s)}\alpha^{(s)}X^* + \theta\left(1-\pi_*^{(s)}\right)\pi_*^{(s)}\alpha^{(s)}X
\end{aligned}$$

so that the denominator is

$$\begin{aligned}
&\sum_{s\in\mathcal{S}_M}\left[\left(\theta\pi_{**}^{(s)}+1\right)\sum_{j:s\in\mathcal{S}_M(j)}\pi_{*j}^{(s)}\alpha^{(s)}X^* + \theta\left(1-\pi_*^{(s)}\right)\pi_*^{(s)}\alpha^{(s)}X\right] = \\
&\sum_{s\in\mathcal{S}_M}\left[\left(1-\pi_*^{(s)}\right)\theta\pi_*^{(s)}\alpha^{(s)}X + \left(1-\pi_{**}^{(s)}\right)\left(\theta\pi_{**}^{(s)}+1\right)\alpha^{(s)}X^*\right]
\end{aligned}$$

Hence,

$$\frac{\tau^{(s)}}{w^*}\frac{dw_*}{d\tau^{(s)}} = -\frac{\left(1-\pi_*^{(s)}\right)\pi_*^{(s)}\alpha^{(s)}X-\left(1-\pi_{**}^{(s)}\right)\pi_{**}^{(s)}\alpha^{(s)}X^*}{\sum_{s\in\mathcal{S}_M}\left[\left(1-\pi_*^{(s)}\right)\pi_*^{(s)}\alpha^{(s)}X+\left(1-\pi_{**}^{(s)}\right)\left(\pi_{**}^{(s)}+1/\theta\right)\alpha^{(s)}X^*\right]}+ \\ +\frac{\sum_{j\in\mathcal{R}}\left[\left(\pi_j^{(s)}+\frac{1}{\theta}\frac{w_j}{X}\frac{\partial X}{\partial w_j}\right)\pi_*^{(s)}\alpha^{(s)}X-\left(1-\pi_{**}^{(s)}\right)\pi_{*j}^{(s)}\alpha^{(s)}X^*\right]\frac{\tau^{(s)}}{w_j}\frac{dw_j}{d\tau^{(s)}}}{\sum_{s\in\mathcal{S}_M}\left[\left(1-\pi_*^{(s)}\right)\pi_*^{(s)}\alpha^{(s)}X+\left(1-\pi_{**}^{(s)}\right)\left(\pi_{**}^{(s)}+1/\theta\right)\alpha^{(s)}X^*\right]}$$

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